Name: **Present group members:**

Worksheet 8-1: Q1
For each problem:

- Sketch the feasible set, is it convex?
- Is the objective function convex? You may use the atoms below.

t^m	convex for $m = 2, 4, 6$
$\|\mathbf{x}\|$	convex
e^t	convex, nondecreasing

- Based on this, state whether the problem is a convex optimization problem or not.

(a)
$$\begin{aligned} \min_{(x,y)} \quad & x^2 - y \\ \text{s.t.} \quad & x^2 + y^2 = 3. \end{aligned}$$

(b)
$$\begin{aligned} \min_{(x,y)} \quad & x + y^4 \\ \text{s.t.} \quad & x + 5y \leq 10. \end{aligned}$$

(c)
$$\begin{aligned} \min_{(x,y)} \quad & -e^{x^2} + y^2 \\ \text{s.t.} \quad & \|(x,y)\| \leq 5. \end{aligned}$$

(d)
$$\begin{aligned} \min_{(x,y)} \quad & (1 + x^2 + x^{10})^2 \\ \text{s.t.} \quad & 5x + 1 \leq 4. \end{aligned}$$

(e)

$$\begin{aligned} \min_{(x,y)} \quad & -x^4 + 3y \\ \text{s.t.} \quad & x + 3y \leq 3. \end{aligned}$$

Worksheet 8-1: Q2

For a position in a company, we need to schedule job interviews for 3 candidates, Alice, Bob, and Charlie, who are coming in that order. The times that each candidate is available for an interview are given by the following table. The goal is to schedule the starting time for each candidate,

Candidate	Times	Interval as hours since 9:00am
Alice	9:00 - 10:30	[0, 1.5]
Bob	10:00 - 11:30	[1, 2.5]
Charlie	9:30 - 12:30	[0.5, 3.5]

denoted t_A, t_B, and t_C for Alice, Bob, and Charlie respectively, such that the minimal starting time difference between consecutive interviews is maximal. *Note that in this problem we won't solve the problem, but we'll walk through how to get this problem into the standard form.*

(a) The objective function is the minimal difference between consecutive starting times of interviews, which can be written as

$$f(t_A, t_B, t_C) = \min\{t_B - t_A, t_C - t_B\}.$$

Write the full optimization problem.

(b) We can pull a mathematical trick by introducing a new variable s to rewrite the optimization problem as

$$\begin{aligned} \max_{t_A, t_B, t_C, s} \quad & s \\ \text{s.t.} \quad & \min\{t_B - t_A, t_C - t_B\} \geq s, \\ & \textit{Insert your constraints from part (a) here.} \end{aligned}$$

Then we can break up the min function. Fill in the blanks below to rewrite the problem without the min function.

$$\begin{aligned} \max_{t_A, t_B, t_C, s} \quad & s \\ \text{s.t.} \quad & t_B - t_A \ \boxed{}\ s, \\ & t_C - t_B \ \boxed{}\ s, \\ & t_A \geq \boxed{} \\ & t_A \leq \boxed{} \\ & t_B \geq \boxed{} \\ & t_B \leq \boxed{} \\ & t_C \geq \boxed{} \\ & t_C \leq \boxed{}\ . \end{aligned}$$

(c) Use the above to write the optimization problem in the standard form

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & A\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq 0 \end{aligned}$$

by defining $\mathbf{x}$, $\mathbf{c}$, A, and $\mathbf{b}$.

Worksheet 8-1: Q3

Consider the following optimization problem:

$$\begin{aligned} \max \quad & 6x + 5y \\ \text{Subject to} \quad & 2x - 3y \leq 5 \\ & x + 4y \leq 11 \\ & 4x + y \leq 15 \\ & x, y \geq 0. \end{aligned}$$

(a) Find **A**, **b**, and **c** such that the problem can be written in the *standard form*

(b) Sketch the feasible set. Where are the extreme points of the feasible set?

(c) Solve the problem by finding the optimal solution.

Worksheet 8-1: Q4

We are planning a football watch party for MSU by making pans of cookies and muffins. A pan of muffins sells for \$6 and a pan of cookies sells for \$10. In addition, we want to make at least as many pans of cookies as pans of muffins. We have only 13 cups of sugar, and a pan of muffins requires 1/2 a cup of sugar and a pan of cookies requires 1 cup of sugar. How many pans of each should we make to minimize the number of pans we need to bake? We want to make at least \$100. Let X be the number of pans of muffins and y be the number of pans of cookies. We want to solve this as a minimization problem.

(a) Write the variables and the objective function.

(b) Write down the constraints as inequalities/equalities.

(c) Recast any inequalities and then write the problem in standard form.

(d) Sketch the feasible region.

(e) Find the minimum of the objective function.