

Name:

Present group members:

Worksheet 1-2: Q1 Circle all the properties of each given set.

1. $S_1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$

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2. $S_2 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$

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3. $S_3 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$

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4. $S_4 = [0, 1) \subset \mathbb{R}$

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5. $S_5 = (0, 1) \cup (2, 3) \subset \mathbb{R}$

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6. $S_6 = [0, 1] \cup [2, 3] \subset \mathbb{R}$

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7. $S_7 = \{(x, y) \in \mathbb{R}^2 : x \geq 0, y \geq 1\}$

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Worksheet 1-2: Q3 Consider the function $f(x_1, x_2) = x_1^2 + 2x_1x_2 + x_2^2$ and the point $\mathbf{x} = (1, 1)$.

1. Compute $\nabla f(\mathbf{x})$ at $\mathbf{x} = (1, 1)$.

2. Write the linear approximation $L(\mathbf{y})$ to f at $\mathbf{x} = (1, 1)$:

$$L(\mathbf{y}) = f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x})$$

3. Use the linear approximation $L(\mathbf{y})$ you just calculated to estimate $f(1.1, 0.9)$.

4. Compute the actual value $f(1.1, 0.9)$.

5. What is the error in the linear approximation? (i.e., $|f(1.1, 0.9) - L(1.1, 0.9)|$)

6. Compute the Hessian matrix $\nabla^2 f(\mathbf{x})$ at $\mathbf{x} = (1, 1)$.

7. For $\mathbf{y} = (1.1, 0.9)$, compute the (approximate) quadratic error term:

$$E_2(\mathbf{y}) \approx \frac{1}{2}(\mathbf{y} - \mathbf{x})^T \nabla^2 f(\mathbf{x})(\mathbf{y} - \mathbf{x})$$

8. For $\mathbf{y} = (1.1, 1.1)$, compute the (approximate) quadratic error term:

$$E_2(\mathbf{y}) \approx \frac{1}{2}(\mathbf{y} - \mathbf{x})^T \nabla^2 f(\mathbf{x})(\mathbf{y} - \mathbf{x})$$

9. What does an error $E_2(\mathbf{y}) = 0$ tell us? What does a non-zero error term tell us? What does the error depend on?

Hint: Take a look at this Desmos to get some intuition: <https://www.desmos.com/3d/esp2pdudke>