

Name:

Present group members:

Worksheet 2-3: Q1

We are going to investigate the function $f(x, y) = 2x^2 - 8xy + y^2$.

1. Write the function in the quadratic form

$$f(x, y) = \mathbf{x}^\top A \mathbf{x} + 2\mathbf{b}^\top \mathbf{x} + c$$

for some symmetric matrix A , vector \mathbf{b} , and scalar c .

2. What is the gradient $\nabla f(x, y)$ and Hessian $\nabla^2 f(x, y)$ of the function? (*Hint: use your matrix A from above.*)

3. Using this, is f coercive? Why or why not?

4. Is f convex? Why or why not?

5. Find and classify the stationary points of $f(x, y) = 2x^2 - 8xy + y^2$. If there are local optima, are they also global?

Worksheet 2-3: Q2

Now consider the function $g(x, y) = 2x^2 - 2xy + y^2 + 6x + 2y$.

1. Write the function in the quadratic form

$$g(x, y) = \mathbf{x}^\top A \mathbf{x} + 2\mathbf{b}^\top \mathbf{x} + c$$

for some symmetric matrix A , vector \mathbf{b} , and scalar c .

2. What is the gradient $\nabla f(x, y)$ and Hessian $\nabla^2 f(x, y)$ of the function? (*Hint: use your matrix A from above.*)

3. Using this, is g coercive? Why or why not?

4. Is g convex? Why or why not?

5. Find and classify the stationary points of $g(x, y) = 2x^2 - 2xy + y^2 + 6x + 2y$. Use the properties of quadratic functions to determine if any local optima are also global optima.