

Name:

Present group members:

Worksheet 12-2: Q1

Consider the problem

$$\begin{aligned} \min \quad & x^2 + y^2 \\ \text{s.t.} \quad & x + y + 1 \leq 0 \end{aligned}$$

1. Does strong duality hold? You need to check whether the problem is convex, and whether the constraints satisfy the generalized Slater's condition.

2. Now, determine q^* using the following steps:

- (a) What is the Lagrangian, $L(x, y)$, and the gradient of the Lagrangian $\nabla_{x,y}L(x, y)$?

- (b) When is the gradient of the Lagrangian zero? Solve for x and y in terms of λ .

(c) For a fixed λ , what is the minimum value of $L(x, y, \lambda)$ over x, y ? Use this to write $q(\lambda) = \inf_{x, y} L(x, y, \lambda)$.

(d) What is $q^* = \max_{\lambda} q(\lambda)$?

3. What do you know about f^* , the optimal value of the primal problem?

Worksheet 12-2: Q2

Consider the problem

$$\begin{aligned} \min \quad & 2x + y \\ \text{s.t.} \quad & -3x + 3y \leq -1 \\ & -3x - 2y \leq -4 \end{aligned}$$

1. Does strong duality hold? You need to check whether the problem is convex, and whether the constraints satisfy the generalized Slater's condition.

2. What are the matrices A , \mathbf{b} , and \mathbf{c} such that the problem can be written as

$$\begin{aligned} \min \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & A\mathbf{x} \leq \mathbf{b} \end{aligned}$$

3. What is the dual problem?

4. Solve the dual problem to find q^* .

5. Use [this desmos](#) to visualize the primal problem. What is f^* , the optimal value of the primal problem? Is it the same as your q^* ?

Worksheet 12-2: Q3

Consider the problem

$$\begin{aligned} \min \quad & -3x - 2y - 1z \\ \text{s.t.} \quad & x + y + z \leq 1 \\ & x - z \leq -2 \end{aligned}$$

1. This is a linear programming problem. What are the matrices A , \mathbf{b} , and \mathbf{c} such that the problem can be written as

$$\begin{aligned} \min \quad & \mathbf{c}^\top \mathbf{x} \\ \text{s.t.} \quad & A\mathbf{x} \leq \mathbf{b} \end{aligned}$$

2. What is the dual problem?

3. Solve the dual problem to find q^* .

4. What is f^* , the optimal value of the primal problem? Justify your answer.