Name: **Present group members:**

Worksheet 6-1: Q1
Consider the points $\mathbf{x}_1 = (2, 3)$ and $\mathbf{x}_2 = (1, 4)$ in $\mathbb{R}^2$.

1. Sketch the points $\mathbf{x}_1$ and $\mathbf{x}_2$ in the plane and draw the line through them. For parts 3-6 below, sketch the point on the line corresponding to each value of λ.

2. Write the equation for the line through the two points using the equation $\lambda\mathbf{x}_1 + (1-\lambda)\mathbf{x}_2 = 0$.

3. What point is on the line when $\lambda = 0$?

4. What point is on the line when $\lambda = 1$?

5. What point is on the line when $\lambda = 0.5$?

6. What point is on the line when $\lambda = -1$?

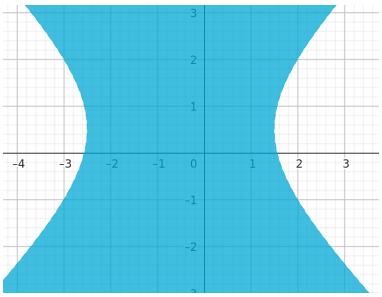
Worksheet 6-1: Q2

For each of the following, indicate whether the given set is convex or not and justify your answer.

i) $S = \{(1,2),(3,4),(4,5),(6,7)\} \subset \mathbb{R}^2$

ii) $S \subseteq \mathbb{R}^2_+ = \{(x_1,x_2) \mid x_1,x_2 \geq 0\}$ where for any $(x,y) \in S$ we have $xy = 0$.

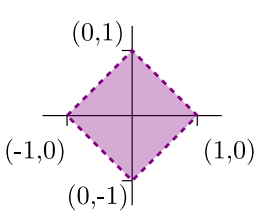
iii) $S = \{\mathbf{x} \in \mathbb{R}^2 \mid x_1^2 - x_2^2 + x_1 + x_2 \leq 4\}$ shown below.



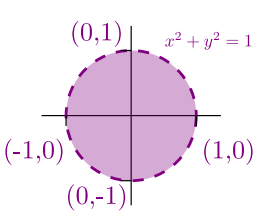
iv) $\{\mathbf{x} : \|x\|_1 \leq 1\}$. L_1-norm: $\|\mathbf{x}\|_1 := \sum_{i=1}^n |x_i|$ in Fig. (a) below.

v) $\{\mathbf{x} : \|x\|_2 \leq 1\}$. L_2-norm: $\|\mathbf{x}\|_2 := \sqrt{\sum_{i=1}^n x_i^2}$ shown in Fig. (b).

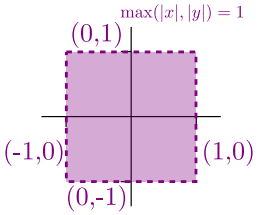
vi) $\{\mathbf{x} : \|x\|_\infty \leq 1\}$. L_∞-norm: $\|\mathbf{x}\|_\infty := \max_i |x_i|$ shown in in Fig. (c).



(a) L_1: $\|x\|_1 = 1$



(b) L_2: $\|x\|_2 = 1$



(c) L_∞: $\|x\|_\infty = 1$

Worksheet 6-1: Q3

The intersection of convex sets is convex, but the **union** of convex sets is not necessarily convex.

1. Draw a simple example to of 2 or 3 convex sets where the union is not convex.

2. Draw a simple example of 2 or 3 convex sets where the union is convex.

Worksheet 6-1: Q4

1. Sketch the points in the set $S = \{(1,1),(2,2),(3,1)\}$ and the convex hull of S in the plane.

2. A convex combination of k points is written as $\lambda_1\mathbf{x}_1 + \lambda_2\mathbf{x}_2 + \ldots + \lambda_k\mathbf{x}_k$ where $\lambda_i \geq 0$ for all i and $\sum_{i=1}^{k} \lambda_i = 1$. What are the λ_1, λ_2, and λ_3 values for each of the following points in the convex hull of S?

   i) $(2,2)$

   ii) $(1,3)$

   iii) $(1.5,1.5)$

   iv) $(2,1)$

3. What points are represented by the following choices of λ_k for convex combinations of points in S?

   (a) $\lambda_1 = \frac{1}{3}$, $\lambda_2 = \frac{1}{3}$, $\lambda_3 = \frac{1}{3}$

   (b) $\lambda_1 = \frac{1}{2}$, $\lambda_2 = \frac{1}{4}$, $\lambda_3 = \frac{1}{4}$

   (c) $\lambda_1 = \frac{1}{2}$, $\lambda_2 = 0$, $\lambda_3 = \frac{1}{2}$

Worksheet 6-1: Q5
Let S contains four points

$$\mathbf{x}_1 = \begin{bmatrix}1\\1\end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix}1\\2\end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix}2\\1\end{bmatrix}, \mathbf{x}_4 = \begin{bmatrix}2\\2\end{bmatrix}.$$

1. Sketch the points in S and the convex hull of S in the plane.

2. How many points in S do we need at most to express any point $\mathbf{x}$ in the convex hull as the convex sum of those points?

3. Given $\mathbf{x} = \begin{bmatrix}1.5\\1.4\end{bmatrix}$, write $\mathbf{x}$ as a convex combination of points in S using the number of points you said you needed above.