Name: **Present group members:**

Worksheet 1-1: Q1

For the L_p norm $\|x\|_p = \sqrt[p]{\sum_{i=1}^{n} |x_i|^p}$, we will check that the restriction $p \geq 1$ is necessary because for $0 \leq p < 1$ the function $\|\cdot\|_p$ is not a norm. Let's investigate this using the case $p = \frac{1}{2}$, where

$$\|x\|_{\frac{1}{2}} = \left(\sum_{i=1}^{n} |x_i|^{\frac{1}{2}}\right)^2.$$

1. Write $\|x\|_{\frac{1}{2}}$ for each of the following vectors x:

   (a) e_1 *(b)* $\begin{bmatrix}1\\1\\0\\1\end{bmatrix}$ *(c)* $\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$ *(d)* $\begin{bmatrix}1\\1\end{bmatrix}$

2. Let's see which of the norm properties (nonnegativity, positive homogeneity, triangle inequality) is violated for $p = \frac{1}{2}$.

   (a) Show that the nonnegativity property is satisfied for an arbitrary x.

   (b) Show that the homogeneity property is satisfied for an arbitrary λ and x.

   (c) Show the triangle inequality is violated. Specifically, consider the two standard basis e_1 and e_2 and check whether $\|e_1 + e_2\|_{0.5} \leq \|e_1\|_{0.5} + \|e_2\|_{0.5}$

Worksheet 1-1: Q2
Let's look a visual representation of planar vectors whose L_p norm is equal to 1.

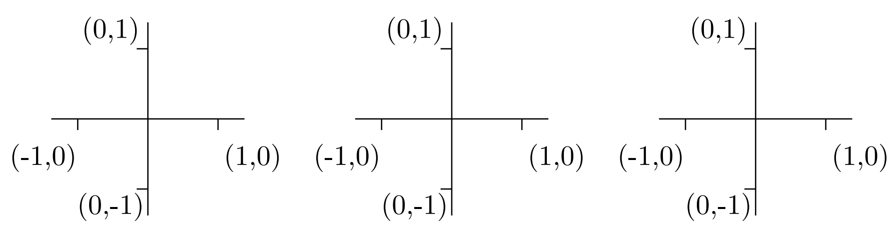
1. Start with L_1 norm. Four vectors that satisfy $\|x\| = 1$ include the vectors $(-1, 0)$, $(0, -1)$, $(1, 0)$, and $(0, 1)$. The points of the arrowhead for each of these vectors are labeled in part (a) of the figure below. For each of the following questions, find the missing x or y entry of the given vector that will make its L_1 norm equal to 1.

   (a) $(0.1, y)$ (b) $(x, 0.5)$ (c) $(0.75, y)$

2. Draw a geometric shape that shows all the possible end points of vectors that satisfy $\|x\|_1 = 1$.

3. Now, let's work the L_2 norm. Write the L_2 norm equation for two arbitrary, planar vectors x and y and set it to 1. Does the resulting equation remind you of the equation of a geometric shape? Draw the resulting geometric shape in part (b) of the figure below.

4. Finally, write the equation that describes $\|x\|_\infty = 1$. What is the requirement on the x y components of a planar vector to satisfy this equation? Draw the geometric shape that represents the arrowheads of all the vectors for which $\|x\|_\infty = 1$.



(a) L_1 (b) L_2 (c) L_∞

Worksheet 1-1: Q3

Recall the Cauchy-Schwarz inequality: For any $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, we have $|\mathbf{x}^\top \mathbf{y}| \leq \|\mathbf{x}\|_2 \cdot \|\mathbf{y}\|_2$. Let's see when we get the equality.

1. The left hand side represents the dot product. Recall $x \cdot y = \|x\|\|y\| \cos\theta$, where θ is the angle between the two vectors. Based on this, when will the two sides of the inequality be equal?

2. Sketch two vectors in $\mathbb{R}^2$ for which the equality holds.