Name: **Present group members:**

For each of the following problems:

(i) *Show that the problem is convex or not convex.*

(ii) *Prove that there exists an optimal solution to the problem.*

(iii) *Determine a solution strategy (e.g., where in the flow chart you need to go) and check any additional requirements in order to apply the KKT conditions (e.g. Slater's condition or regularity).*

(iv) *Are the KKT conditions necessary? Sufficient?*

Once you have done the above for each problem, go back and use the KKT conditions to find the optimal solution of each problem. Note you will often need to use cases ($\lambda_i = 0$ *vs.* $\lambda_i > 0$*) to find the KKT points. Use Desmos to check your answer.*

1.

$$\begin{aligned} \min \quad & x_1^4 - 2x_2^2 - x_2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 + x_2 \le 0 \end{aligned}$$

Hint: the constraint can also be written as $x_1^2 + \left(x_2 + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 \le 0$.

2.

$$\begin{aligned} \min \quad & 2x_1 + x_2 \\ \text{s.t.} \quad & 4x_1^2 + x_2^2 - 2 \le 0 \\ & 4x_1 + x_2 + 3 \le 0 \end{aligned}$$

3.

$$\begin{aligned} \min \quad & x_1^3 + x_2^3 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 1 \end{aligned}$$

4.

$$\begin{aligned} \min \quad & x_1^4 - x_2^2 \\ \text{s.t.} \quad & x_1^2 + x_2^2 \leq 1 \\ & 2x_2 + 1 \leq 0 \end{aligned}$$