Name: **Present group members:**

Worksheet 7-3: Q1

Consider $\max\{\mathbf{x}^\top Q\mathbf{x} \mid \|\mathbf{x}\| \le 1\}$ where $\mathbf{x} \in \mathbb{R}^n$ and $Q \succeq 0$.

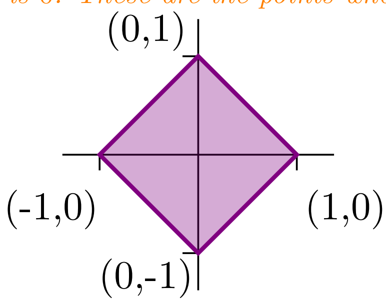
(a) Does a maximizer exist? Justify by checking all the conditions of the relevant theorem.

- *The theorem says that if the function is continuous and convex defined over a convex compact set, then a maximizer exists. Further, it occurs at the extreme points of the set.*
- *The function $\mathbf{x}^\top Q\mathbf{x}$ is continuous and convex since $Q \succeq 0$.*
- *The set $\{\mathbf{x} \mid \|\mathbf{x}\| \le 1\}$ is convex and compact. It actually doesn't matter which norm we choose, this will always be true. More on options for norms in the next question.*
- *Since it satisfies the requirements of the theorem, a maximizer exists.*

(b) Let $\mathbf{x} \in \mathbb{R}^2$ (that is, assume $n = 2$ above) and answer the following questions.

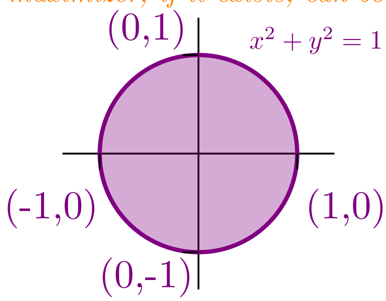
(i) Using L_1-norm for $\mathbf{x}$: Sketch the feasible region and point out where a maximizer, if it exists, can be found.

The region where $\|\mathbf{x}\|_1 \le 1$ is the diamond-shaped region shown in the figure. The extreme points of this set are the points where one of the coordinates is ± 1 and the other is 0. These are the points where a maximizer, if it exists, can be found.



(ii) Using L_2-norm for $\mathbf{x}$: Sketch the feasible region and point out where a maximizer, if it exists, can be found.

The region where $\|\mathbf{x}\|_2 \le 1$ is the circular region shown in the figure. The extreme points of this set are the points on the boundary of the circle. These are the points where a maximizer, if it exists, can be found.



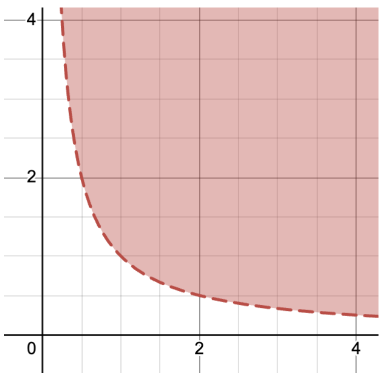
Worksheet 7-3: Q2
Consider the function

$$f : [0, \infty) \to \mathbb{R} \cup \{\infty\}$$
$$x \mapsto \begin{cases} \infty & \text{if } x = 0 \\ 1/x & \text{if } x > 0 \end{cases}$$

- What is the effective domain of f, $\text{dom}(f)$?

  The effective domain of f is $\text{dom}(f) = (0, \infty)$ since $f(x) = \infty$ for $x = 0$ and $f(x) = 1/x$ for $x > 0$.

- Sketch the epigraph of f.



- Is f convex? Justify your answer. *The function f is convex because the effective domain of f is $(0, \infty)$ and $f(x) = 1/x$ is convex on this domain. This can be shown by computing the second derivative of f:*

$$f''(x) = \frac{2}{x^3} > 0 \text{ for } x > 0.$$

  Since the second derivative is positive for all $x > 0$, f is convex on its effective domain. Additionally, since $f(x) = \infty$ for $x = 0$, the function is convex on the entire domain $[0, \infty)$.

Worksheet 7-3: Q3

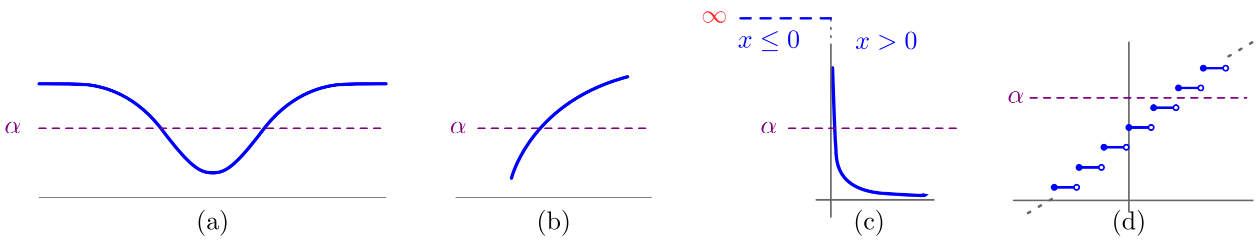
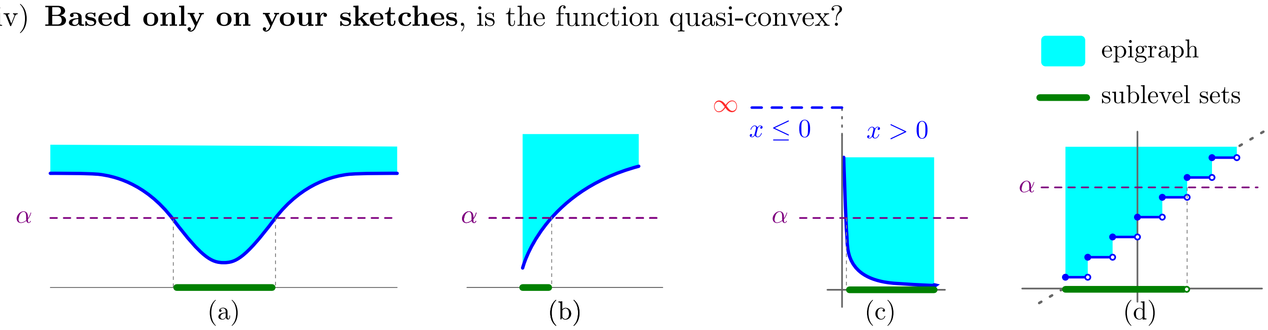


Figure 1: Problem 3

For each of the functions shown in the figure, answer the following:

(i) Sketch the sublevel sets at level α.
(ii) Sketch the epigraph.
(iii) **Based only on your sketches**, is the function convex? Justify your answer.
(iv) **Based only on your sketches**, is the function quasi-convex?



(i) Not convex, but it is quasi-convex.
(ii) Not convex but quasiconvex.
(iii) Convex (which also implies quasiconvex).
(iv) Not convex, but it is quasiconvex.