Name: **Present group members:**

Worksheet 7-1: Q1

Show that $f(t) = t^2$ is convex directly using the definition of a convex function. *Hint: expand the square and use the fact that* $\lambda(1-\lambda) \geq 0$ *for* $\lambda \in [0,1]$.

For the rest of this worksheet, you can assume that the following "atom" functions are convex.

$f(t) = mt + b$ for any $m, b \in \mathbb{R}$. $f(t) = t^2$ $f(t) = e^t$

$f(t) = t^4$ $f(t) = -\ln(t)$ for $t > 0$

Worksheet 7-1: Q2

Show that each of the following functions is convex. Use the theorems from the lecture to justify your answer.

(a) $f(x, y) = x^2 + 3y^2$

(b) $h(x, y) = \exp(x + y)$.

(c) $f(x, y) = x^2 + 2xy + 3y^2 + 2x - 3y$.

(d) $f(x, y, z) = \exp(x - y + z) + \exp 2y + x$.

(e) $f(x, y) = -\ln(xy)$, $x > 0, y > 0$.

Worksheet 7-1: Q3

Are the following functions convex? You must justify your answer.

(a) $f(x, y) = xy$.

(b) $\|\mathbf{x}\|$ convex for any norm on $\mathbb{R}^n$. Use the definition of convex function for this part.

(c) $h(\mathbf{x}) = \exp(\|\mathbf{x}\|)$.

(d) $\|\mathbf{x}\|^2$ for any norm on $\mathbb{R}^n$.