Name: **Present group members:**

Worksheet 6-3: Q1
Consider the optimization problem $Ax = b$ given by

$$\begin{bmatrix} 1 & 5 & 3 & 4 & 6 \\ 0 & 1 & 3 & 5 & 6 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 14 \\ 7 \end{bmatrix}$$

(a) Which subsets of 2 columns of the matrix A are linearly independent? (Hint: there are 10 such subsets.)

(b) Consider the point $\mathbf{x} = (0, 2, 0, 1, 0)$. Check that this a solution to the optimization problem $Ax = b$.

(c) Is the point $\mathbf{x} = (0, 2, 0, 1, 0)$ a basic feasible solution? Why or why not?

Worksheet 6-3: Q2

Consider the optimization problem $Ax = b$ given by

$$\begin{bmatrix} 1 & 7 & 22 & 0 & 5 & 0 & 1 \\ 0 & 3 & -5 & 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 1 & -4 & 0 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}$$

(a) Check that the set of the 1st, 4th, and 6th columns of A are linearly independent.

(b) Find a basic feasible solution to the optimization problem $Ax = b$ that corresponds to the 1st, 4th, and 6th columns of A. That is, find a solution such that the only nonzero entries of $\mathbf{x}$ are the 1st, 4th, and 6th entries.

For the same optimization problem,

$$\begin{bmatrix} 1 & 7 & 22 & 0 & 5 & 0 & 1 \\ 0 & 3 & -5 & 0 & 2 & 1 & 2 \\ 0 & 1 & 0 & 1 & -4 & 0 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}$$

(c) Check that the 1st, 3rd, and 4th columns of A are linearly independent.

(d) Find a basic feasible solution to the optimization problem $Ax = b$ that corresponds to the 1st, 3rd, and 4th columns of A. That is, find a solution such that the only nonzero entries of $\mathbf{x}$ are the 1st, 3rd, and 4th entries.

Worksheet 6-3: Q3

Let S be a closed, bounded, and covex set. Figure 1a shows a sampling of points from S that includes all the extreme points and some interior points. Answer the following questions.

(a) Mark the extreme points ext(S) on Fig. 1b.

(b) Highlight the region that represents the convex hull of the extreme points conv(ext(S)) on Fig. 1c.

(c) What is your best guess on what the full set S looks like? Explain your answer.

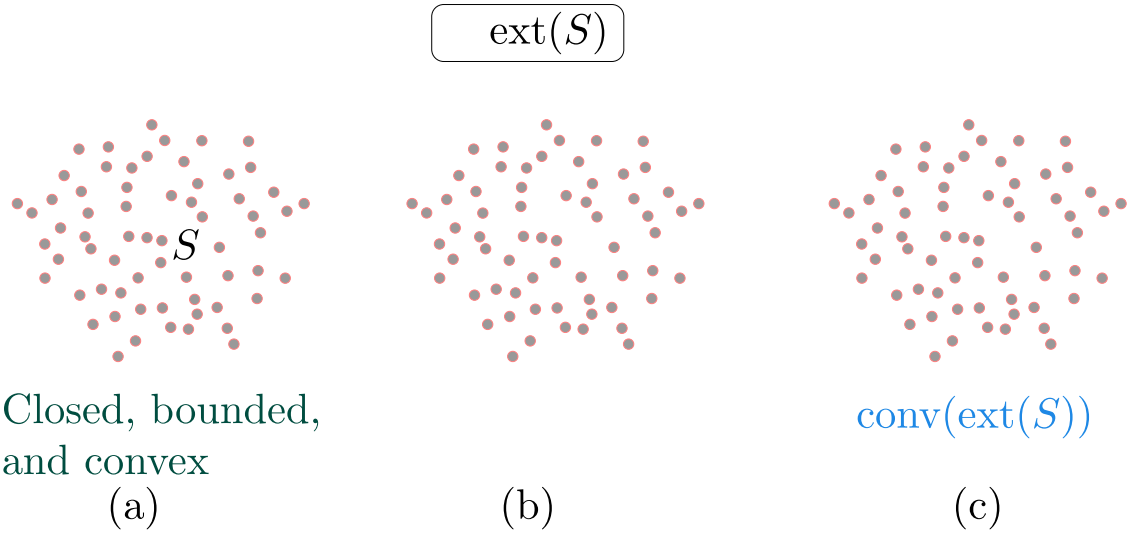

Figure 1: Problem 2.