Name: **Present group members:**

Worksheet 1-2: Q1

For the matrix $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, find

1. The induced 1-norm, $\|A\|_1$.

2. The induced ∞-norm, $\|A\|_\infty$.

3. The Frobenius norm, $\|A\|_F$.

Worksheet 1-2: Q2

We are going to determine the induced 2-norm for the matrix $A = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$.

1. Compute $A^T A$.

2. Find the eigenvalues of $A^T A$. We generally sort these in decreasing order so that $\lambda_1 \geq \lambda_2$.

3. Find the singular values of A.

4. Compute the induced 2-norm of A.

Worksheet 1-2: Q3

1. Show that for $A \in \mathbb{R}^{m \times n}$, if λ is a non-zero eigenvalue of $A^T A$ with eigenvector v, then λ is a non-zero eigenvalue of AA^T with eigenvector Av.

2. Use the previous statement to show that for $A \in \mathbb{R}^{m \times n}$, $\|A\| = \|A^T\|$ for the spectral norm (AKA induced 2-norm).

3. Show that $\|A\|_F^2 = \sum_{i=1}^{n} \lambda_i(A^T A)$. *(Hint: Consider the trace $A^T A$.)*