Name: **Present group members:**

Some useful facts about Lipschitz functions:

- The Lipshitz requirement can be reframed as $\frac{|f(\mathbf{x})-f(\mathbf{y})|}{\|\mathbf{x}-\mathbf{y}\|} \leq L$, and that thing on the left is related to the derivative. That means that if the derivative is bounded, then the function is Lipschitz with constant L.
- If $f : U \subset \mathbb{R}^n \to \mathbb{R}$, and $g : [a, b] \subset \mathbb{R} \to \mathbb{R}$ are Lipschitz continuous with constants L_1, and L_2, respectively, then their composition $g \circ f = g(f)$ is Lipschitz continuous with constant L_1L_2.
- Lipschitz functions are closed under common algebraic operations: addition, subtraction, and scalar multiplication. This means, for example, if you add two Lipshitz functions then the resulting function from these operations is also Lipschitz.

Worksheet 4-3: Q1

For each of the following state (with justification):

(a) If the function is Lipschitz (and if so what is L)
(b) If the derivative is Lipschitz (and if so, what is L),
(c) If the function is in $C_L^{1,1}$ (and if so, what is L).

1. $f(x) = mx + b$ (linear function)

2. $f(x) = \sqrt{x}$

3. $f(x) = x^2$

4. $f(x) = \sin(x)$

5. $f(x) = \exp(-x)$ for $x \in [0, \infty)$

6. $f(x) = 2\sin(x) - 10.9y^2 + \pi \exp(-(x^2 + y^2))$

Worksheet 4-3: Q2

Given the function $f(x, y) = x^2 + y^4$, answer the following questions.

i) Write down the first 3 iterations of the gradient descent algorithm starting at $x_0 = (1, 1)$ using constant step size $t = \frac{1}{2}$.

ii) Comment on the results, are the iterations converging?

iii) Check whether the function satisfies the assumptions of the gradient method convergence theorem, and use your analysis to comment on the results of the iterations.