Ch 9.2: Support Vector Classifier

Lecture 27 - CMSE 381

Michigan State University
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Dept of Computational Mathematics, Science & Engineering

Wed, Apr 1, 2026

Announcements

Last time:

- 9.1 Maximal Margin Classifier

This lecture:

- 9.2 Support Vector Classifier

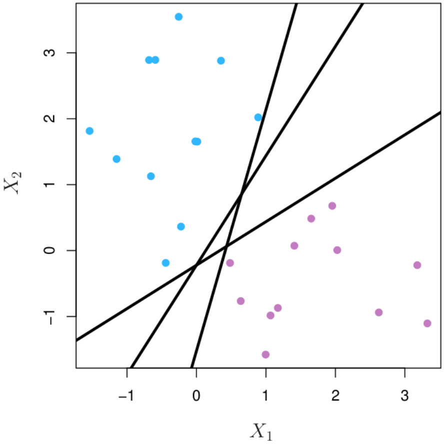
Announcements:

21	W	3/18	Polynomial & Step Functions	7.1-7.2		
22	F	3/20	Step Functions; Basis functions; Start Splines	7.2-7.4		
23	M	3/23	Regression Splines	7.4		
24	W	3/25	Decision Trees	8.1		Q7
25	F	3/27	Random Forests	8.2.1, 8.2.2	HW #5 Due Sun 3/29	
26	M	3/30	Maximal Margin Classifier	9.1		
27	W	4/1	SVC	9.2		Q8
28	F	4/3	SVM	9.3, 9.4		
29	M	4/6	Single Layer NN	10.1		
30	W	4/8	Multi Layer NN	10.2		Q9
31	F	4/10	CNN	10.3	HW #6 Due Sun 4/12	
32	M	4/13	Unsupervised learning / clustering	12.1, 12.4		
33	W	4/15	Virtual: Project Office Hours			Q10
	F	4/17	***Review***			
	M	4/20	**Midterm #3**			
	W	4/22				
	F	4/24			**Project Due**	

Section 1

Last time

Separating Hyperplane



Require that for every data point:

$$\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} > 0 \text{ if } y_i = 1$$
$$\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} < 0 \text{ if } y_i = -1$$

Equivalently

Require that for every data point

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) > 0$$

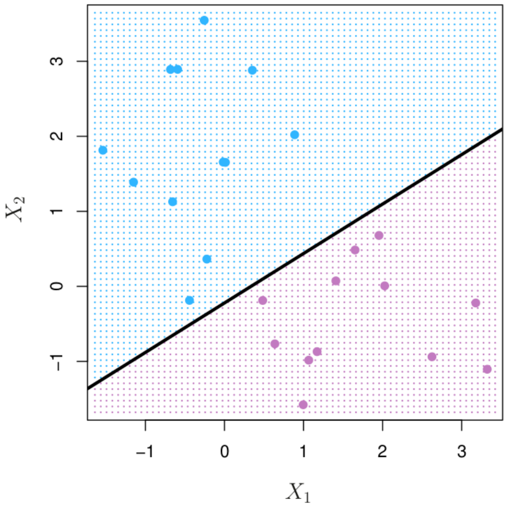
Separating hyperplane becomes a classifier

If you have a separating hyperplane:

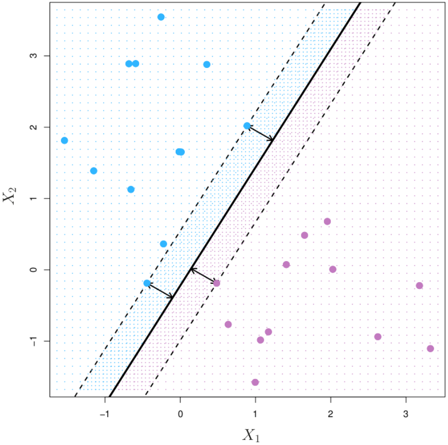
- Check

$$f(x^*) = \beta_0 + \beta_1 x_1^* + \beta_2 x_2^* + \cdots + \beta_p x_p^*$$

- If positive, assign $\hat{y} = 1$
- If negative, assign $\hat{y} = -1$



Maximal margin classifier



- For a hyperplane, the *margin* is the smallest distance from any data point to the hyperplane.
- Observations that are closest are called *support vectors*.
- The *maximal margin hyperplane* is the hyperplane with the largest margin
- The classifier built from this hyperplane is the *maximal margin classifier*.

Test your understanding: PollEv

Mathematical Formulation

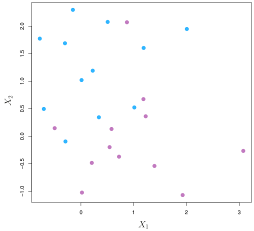
$$\underset{\beta_0,\beta_1,\ldots,\beta_p,M}{\text{maximize}}\; M$$

$$\text{subject to } \sum_{j=1}^{p} \beta_j^2 = 1,$$

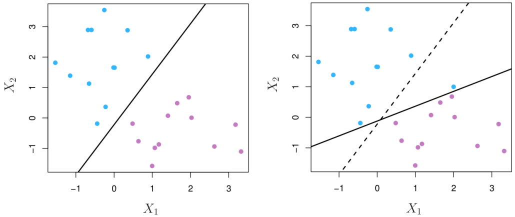
$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M \;\; \forall\, i = 1, \ldots, n$$

Problems

Might be no separating hyperplane



Sensitivity to new points



What will you learn today?

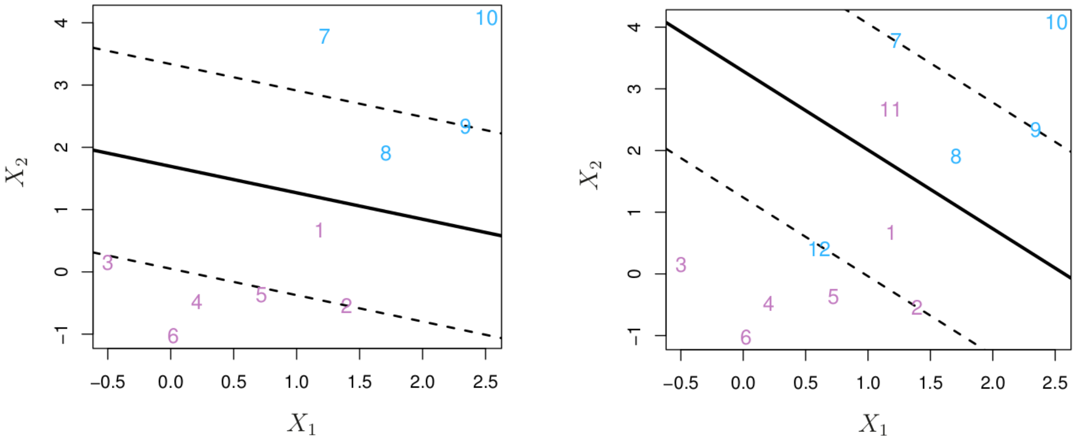
- What is the difference between Support Vector Classifier and Maximal Margin Classifier?
  - What short-comings of Maximal Margin Classifier does Support Vector Classifier overcome?
  - How?
  - You should be able to describe the difference verbally, graphically, and mathematically.
- How to interpret each element of the mathematical formulation of Support Vector Machine graphically?
  - Given a graph of Support Vector Classifier and some data points, you should be able to derive the values of different parameters in the mathematical formulation (e.g. M, ϵ_i).
- What parameters controls the flexibility (hence bias-variance trade-off) of Support Vector Classifier?
- What are the support vectors of Support Vector Machine?

Section 2

Support Vector Classifier

Basic idea

Soft margin



10
7
9
8
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6
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3
2
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0
−1
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
X_2
X_1
10
7
11
9
8
1
12
3
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4
3
2
1
0
−1
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
X_2
X_1

Mathematical Formulation of SVC

$$\underset{\beta_0,\beta_1,\ldots,\beta_p,\epsilon_1,\ldots,\epsilon_n,\,M}{\text{maximize}} \quad M$$

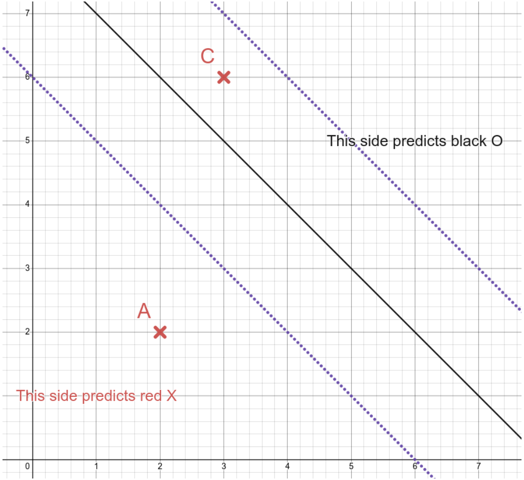
$$\text{subject to} \;\; \sum_{j=1}^{p} \beta_j^2 = 1,$$

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M(1 - \epsilon_i),$$

$$\epsilon_i \geq 0, \;\; \sum_{i=1}^{n} \epsilon_i \leq C,$$

Find positive ε's that will satisfy this

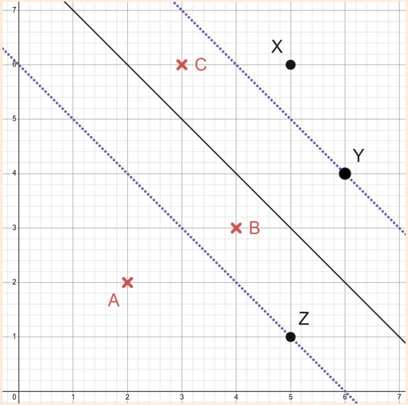
Fix $M = \sqrt{2}$ $\qquad y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M(1 - \varepsilon_i)$



What is ε?

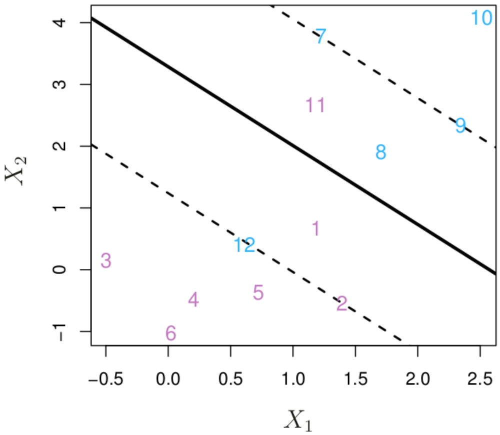
$$\text{Fix } M = \sqrt{2} \qquad y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M(1 - \varepsilon_i)$$

Fill in the table so that the inequality is satisfied.



Point	Left Side	ε_i	$M(1-\varepsilon_i)$
A	$2\sqrt{2}$	0	$\sqrt{2}$
B	$\frac{\sqrt{2}}{2}$		
C	$-\frac{\sqrt{2}}{2}$	1.5	$-\frac{\sqrt{2}}{2}$
X	$\frac{3\sqrt{2}}{2}$		
Y	$\sqrt{2}$		
Z	$-\sqrt{2}$		

What is ε?



What is C?

$$\underset{\beta_0,\beta_1,\ldots,\beta_p,\epsilon_1,\ldots,\epsilon_n,\, M}{\text{maximize}} \quad M$$

$$\text{subject to} \ \sum_{j=1}^{p} \beta_j^2 = 1,$$

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M(1 - \epsilon_i),$$

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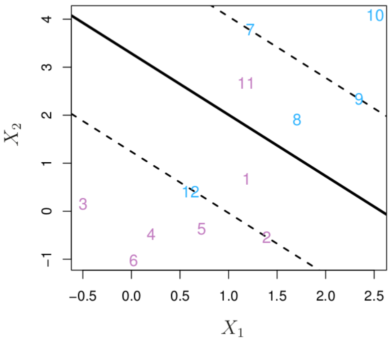
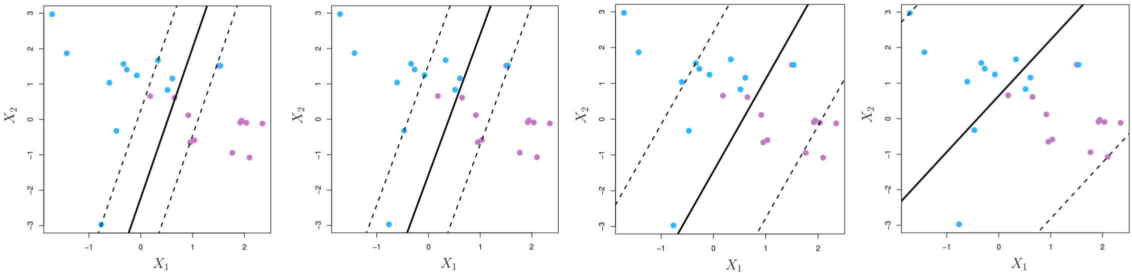
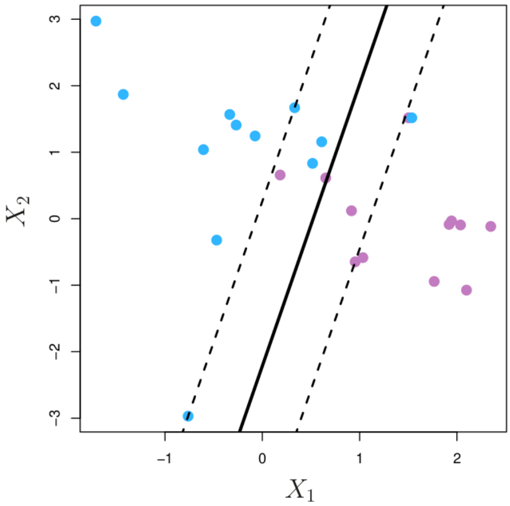


Examples messing with C



Increasing C $\rightarrow$

What affects the hyperplane?



Coding

Maximal Margin Classifier

$$\underset{\beta_0,\beta_1,\ldots,\beta_p,M}{\text{maximize}} \; M$$

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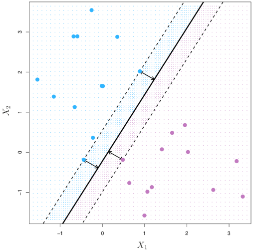

Support Vector Classifier

$$\underset{\beta_0,\beta_1,\ldots,\beta_p,\epsilon_1,\ldots,\epsilon_n,\,M}{\text{maximize}} \quad M$$

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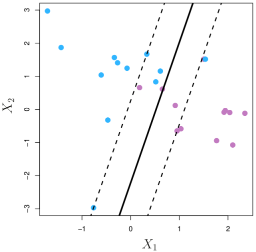

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