Ch 10.3: Convolutional Neural Nets

Lecture 31 - CMSE 381

Michigan State University
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Dept of Computational Mathematics, Science & Engineering

Fri, Apr 10, 2026

Last time:

- Multilayer NN
- pyTorch

This lecture:

- CNNs

Final countdown:

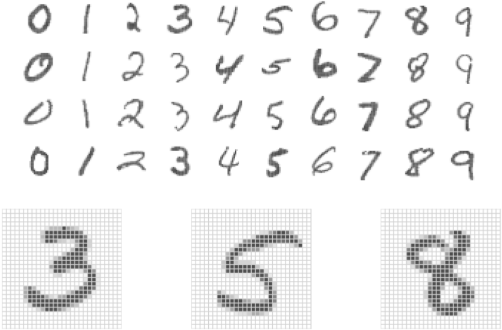
- HW #6 is due Sunday 4/12
- Exam 3 is 4/20
- Project is due 4/24

21	W	3/18	Polynomial & Step Functions	7.1-7.2		
22	F	3/20	Step Functions; Basis functions; Start Splines	7.2-7.4		
23	M	3/23	Regression Splines	7.4		
24	W	3/25	Decision Trees	8.1		Q7
25	F	3/27	Random Forests	8.2.1, 8.2.2	HW #5 Due Sun 3/29	
26	M	3/30	Maximal Margin Classifier	9.1		
27	W	4/1	SVC	9.2		Q8
28	F	4/3	SVM	9.3, 9.4		
29	M	4/6	Single Layer NN	10.1		
30	W	4/8	Multi Layer NN	10.2		Q9
31	F	4/10	CNN	10.3	HW #6 Due Sun 4/12	
32	M	4/13	Unsupervised learning / clustering	12.1, 12.4		
33	W	4/15	Virtual: Project Office Hours			Q10
	F	4/17	***Review***			
	M	4/20	**Midterm #3**			
	W	4/22				
	F	4/24			**Project Due**	

Section 1

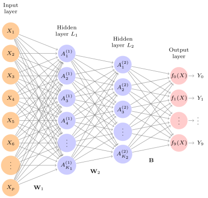
Last time: Neural Nets

MNIST



- Goal: Build a model to classify images into their correct digit class
- Each image has $p = 28 \cdot 28 = 784$ pixels
- Each pixel is grayscale value in [0,255]
- Data converted into column order
- Output represented by one-hot vector $Y = (Y_0, Y_1, \cdots, Y_9)$
- 60K training images, 10K test images

Neural network architecture for MNIST



$$
\begin{aligned}
A_k^{(1)} &= h_k^{(1)}(X) \\
&= g(w_{k0}^{(1)} + \textstyle\sum_{j=1}^{p} w_{kj}^{(1)} X_j)
\end{aligned}
$$

$$
\begin{aligned}
A_\ell^{(2)} &= h_\ell^{(2)}(X) \\
&= g(w_{\ell 0}^{(2)} + \textstyle\sum_{k=1}^{K_1} w_{\ell k}^{(2)} A_k^{(1)})
\end{aligned}
$$

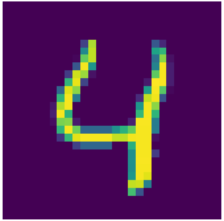
$$
\begin{aligned}
Z_m &= \beta_{m0} + \textstyle\sum_{\ell=1}^{K_2} \beta_{m\ell} h_\ell^{(2)}(X) \\
&= \beta_{m0} + \textstyle\sum_{\ell=1}^{K_2} \beta_{m\ell} A_\ell^{(2)},
\end{aligned}
$$

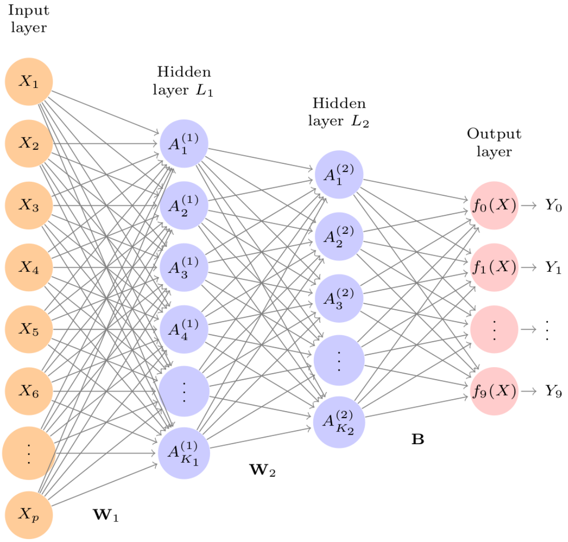
$$
f_m(X) = \Pr(Y = m|X) = \frac{e^{Z_m}}{\sum_{\ell=0}^{9} e^{Z_\ell}},
$$

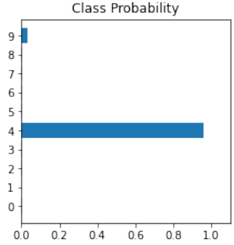
- Two hidden layers.
- Softmax for classification output
- We used L_1 has 128 units; L_2 has 64
- 10 output variables due to class labeling
- Result is we are training approx 110K weights

Test your understanding: PollEv

MNIST learning







What will you learn today?

- What is the architecture of a convolutional neural network?
  - You should be able to describe the types of layers, how they are arranged, and the purpose of each of them.
- How does convolution work?
  - You should be able to compute the result of the convolution between a simple matrix and a simple filter by hand.
- What information does Convolutional Neural Network capture through convolution?

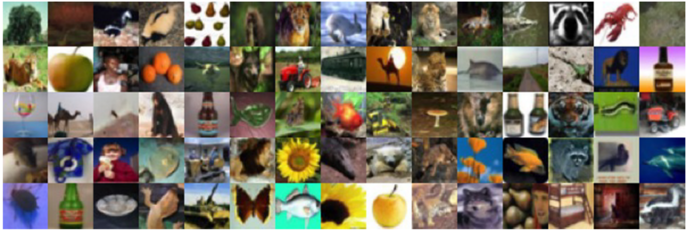
Section 2

Convolutional Neural Network

Last time: Flattening the image – why is it not great?

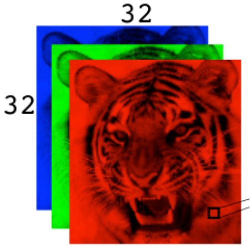
$$\begin{pmatrix} 1 & 1 & 0 \\ 4 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \\ 4 \\ 2 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}$$

Example data set: CIFAR100 Data

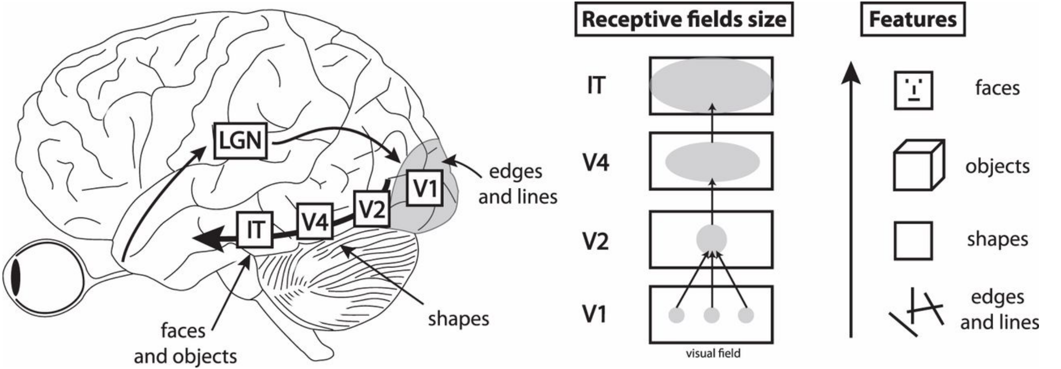


- 60,000 images: 50K training, 10K test
- Labels with 20 super classes (e.g. aquatic mammals)
- 5 classes per super class (beaver, dolphin, otter, seal, whale)
- Images are 32x32

Image channel data

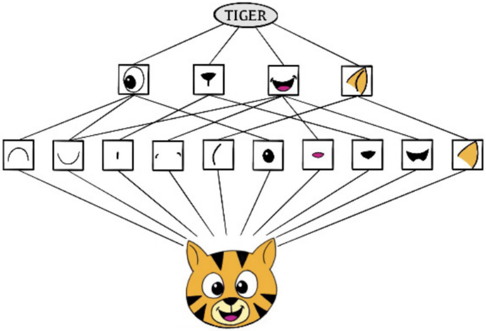


How does your brain do it? The visual hierarchy



Mauro Manassi, Bilge Sayim, Michael H. Herzog; When crowding of crowding leads to uncrowding. Journal of Vision 2013;13(13):10. https://doi.org/10.1167/13.13.10.

CNNs



Convolution layer

Convolution Filter

Original Image:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$$

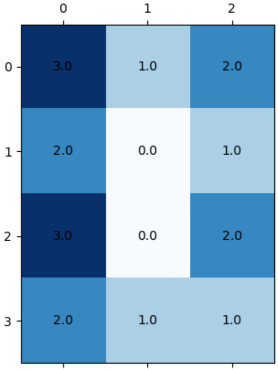
Convolution filter:

$$\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$$

Convolved Image

$$\begin{bmatrix} a\alpha + b\beta + d\gamma + e\delta & b\alpha + c\beta + e\gamma + f\delta \\ d\alpha + e\beta + g\gamma + h\delta & e\alpha + f\beta + h\gamma + i\delta \\ g\alpha + h\beta + j\gamma + k\delta & h\alpha + i\beta + k\gamma + l\delta \end{bmatrix}$$

Convolution Filter Example



Filter:

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

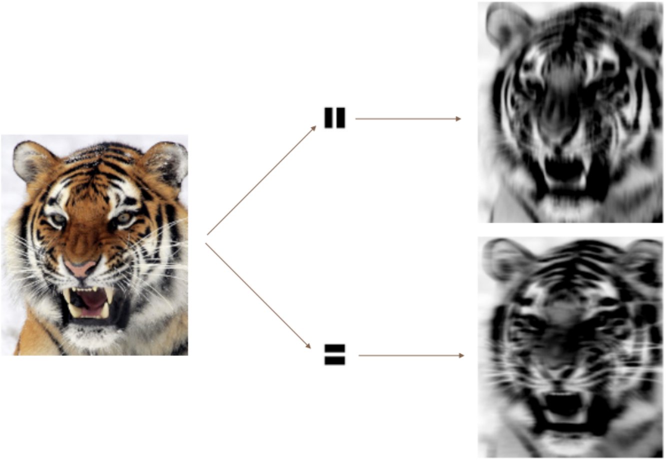
Same example, different filter

What is the convolved image?

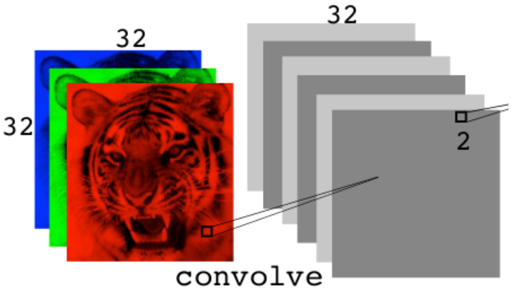
Filter:

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Convolution filter: Bigger example



Convolution layer

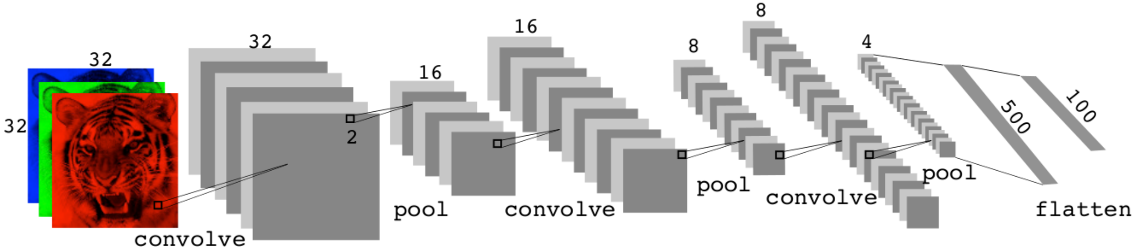


More notes on convolution

Pooling layers

$$\text{Max pool} \begin{bmatrix} 1 & 2 & 5 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 1 & 3 & 4 \\ 1 & 1 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

Putting it together to make a CNN

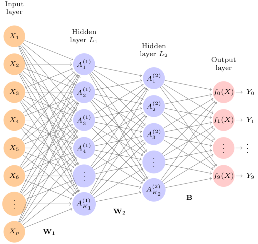


https://poloclub.github.io/cnn-explainer/

Coding

TL;DR

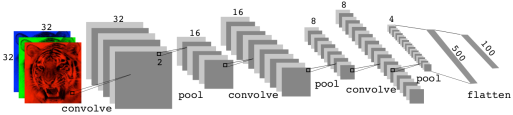
Feed Forward Neural Net



$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$

- Combines input data using learned weights
- Linear combo of those to get output
- Sometimes softmax to get probability of classification

CNN



- Specialized NN
- Gets next layer via
  - Convolution layer
  - Pooling Layer
  - Fully connected layer

Next time

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