# Ch 10.1: Neural Nets

## Lecture 29 - CMSE 381

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::
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Mon, Apr 7, 2025

# Announcements

**Last time:**

- SVM

**This lecture:**

- Feed Forward Neural Nets

**Announcements:**

- Homework #9 due Sunday, Apr 13
- No class Wednesday Apr 16. Virtual office hours by request.

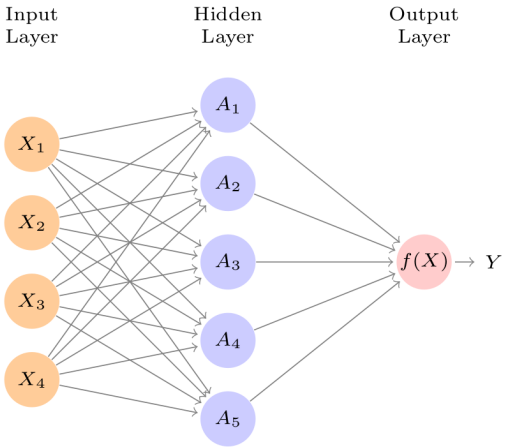
| | | | | | | |
|---|---|---|---|---|---|---|
| 21 | W | 3/19 | Polynomial & Step Functions | 7.1-7.2 | | |
| 22 | F | 3/21 | Step Functions; Basis functions; Start Splines | 7.2-7.4 | | |
| 23 | M | 3/24 | Regression Splines | 7.4 | | |
| 24 | W | 3/26 | Decision Trees | 8.1 | HW #6 Due Wed 3/26 | Q7 |
| 25 | F | 3/28 | Random Forests | 8.2.1, 8.2.2 | HW #7 Due Sun 3/30 | |
| 26 | M | 3/31 | Maximal Margin Classifier | 9.1 | | |
| 27 | W | 4/2 | SVC | 9.2 | | Q8 |
| 28 | F | 4/4 | SVM | 9.3, 9.4 | HW #8 Due Sun 4/6 | |
| 29 | M | 4/7 | Single Layer NN | 10.1 | | |
| 30 | W | 4/9 | Multi Layer NN | 10.2 | | Q9 |
| 31 | F | 4/11 | CNN | 10.3 | | |
| 32 | M | 4/14 | Unsupervised learning / clustering | 12.1, 12.4 | HW #9 Due Sun 4/13 | Q10 |
| 33 | W | 4/16 | Virtual: Project Office Hours | | | |
| | F | 4/18 | ***Review*** | | | |
| | M | 4/21 | **Midterm #3** | | | |
| | W | 4/23 | | | | |
| | F | 4/25 | | | **Project Due** | |
| | | | | | | |
| | | | | | | |
| | | | **No final exam** | | | |

Section 1

# Neural Nets

# The idea

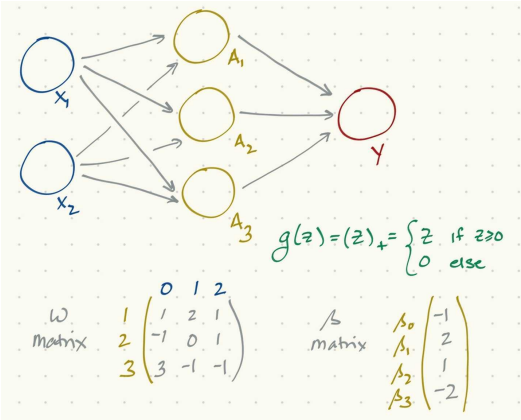
# Feed Forward Neural Network: The cartoon



# A very simple example

## Computing $A_k$ for $(1, 0)$

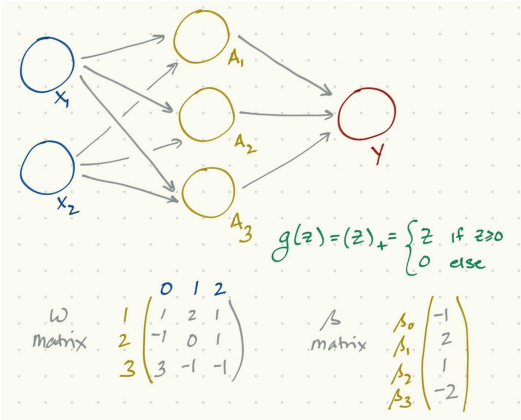
$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$



# A very simple example

## Computing $Y$ for $(1, 0)$

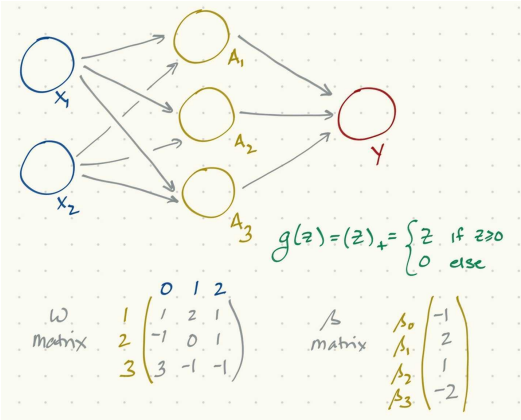
$$f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$



# A very simple example

## Computing $Y$ for $(0, 1)$

$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j), \qquad f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$



# A different example

- Draw the diagram for a neural net with input data points with $p = 3$ (i.e., $(X_1, X_2, X_3)$) and two units in the hidden layer.
- Using the $\omega$ and $\beta$ matrices, what is the output predicted $Y$ for the point $(2, 0, 1)$?

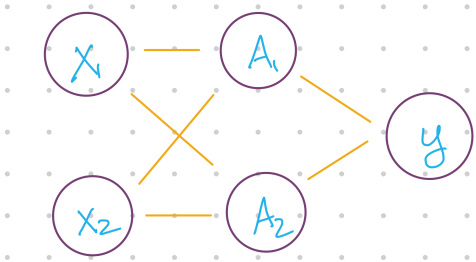
$$\omega = \begin{pmatrix} 1 & 0 & -2 & 2 \\ -3 & 1 & 0 & -1 \end{pmatrix} \qquad \beta = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

- Use the activation function

$$g(z) = (z)_+ = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{else.} \end{cases}$$

# Extra space

# What if there's no activation function?




$$\omega = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \qquad \beta = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

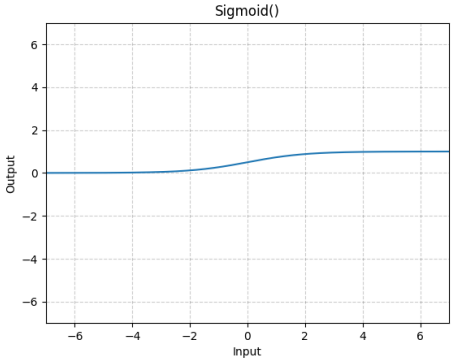
# Choices for activation function

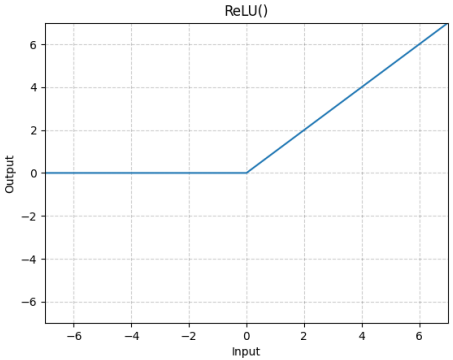
Sigmoid:

$$g(z) = \frac{e^z}{1+e^z} = \frac{1}{1+e^{-z}}$$

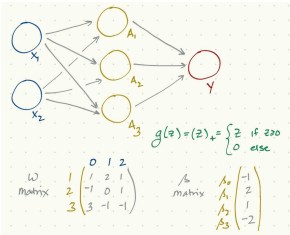
ReLU: Rectified linear unit

$$g(z) = (z)_+ = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{else.} \end{cases}$$



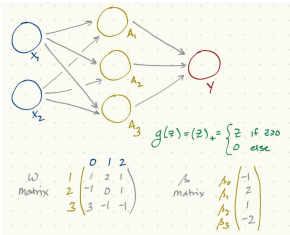
# Matrix version: First layer




$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$

$$A = g(\mathbf{W} \cdot \mathbf{X}) \qquad \mathbf{X}^T = (1\ X_1\ X_2\ \cdots\ X_p)$$

# Matrix version: Output




$$f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$

$$Y = \beta \cdot \mathbf{A} \qquad \mathbf{A}^T = (1\ A_1\ A_2\ \cdots\ A_K)$$

# Now what?

Choose parameters by minimizing RSS, $\sum_{i=1}^{n}(y_i - f(x_i))^2$

**Chosen in advance:**

**Tuned by the model:**

# Coding

# Next time

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| | | | **No final exam** | | | |