

Ch 2.2: Assessing Model Accuracy

Lecture 3 - CMSE 381

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Fri, Jan 17, 2025

Announcements:

- Last time:
- Ch 2.1, Vocab day!

- Get on slack!
 - ▶ +1 point on the first homework if you post a gif in the thread
- First homework due Sunday, 1/26. Covers:
 - ▶ Mon 1/13 lecture
 - ▶ Weds 1/15 Lecture
 - ▶ Today 1/17 Lecture
- Office hours
 - ▶ Tu-W, Dr. Bao (EGR 2507L) 9am - 10 am
 - ▶ Wednesdays, Christy (EGR TBD) 2:30pm - 4:00pm
 - ▶ Thursdays, Christy (EGR TBD) 8:45am - 10:15am

Covered in this lecture

- Mean Squared Error (regression)
- Train vs Test
- Bias Variance Trade off

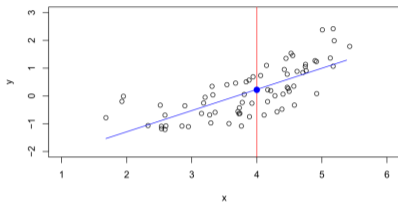
Quick review of notation

Section 1

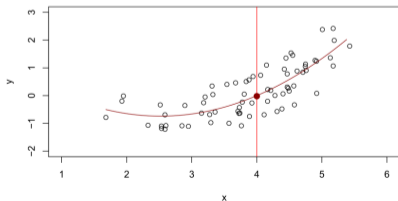
Mean Squared Error

Which is better?

A linear model $\hat{f}_L(X) = \hat{\beta}_0 + \hat{\beta}_1 X$ gives a reasonable fit here



A quadratic model $\hat{f}_Q(X) = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 X^2$ fits slightly better.

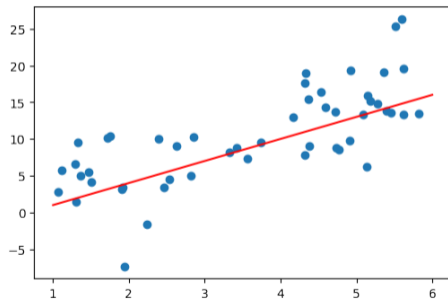


No free lunch

Mean Squared Error

Error in the regression setting

$$MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{f}(x_i))^2$$



Group Work

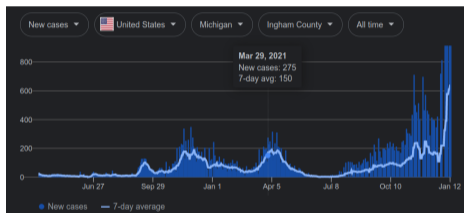
Given the following data, you decide to use the model

$$\hat{f}(X_1, X_2) = 1 - 3X_1 + 2X_2.$$

What is the MSE?

X_1	X_2	Y
0	7	14
1	-3	-6
5	2	-10
-1	1	7

Training MSE



Training set:

The observations

$\{(x_1, y_1), \dots, (x_n, y_n)\}$ used to get
the estimate \hat{f}

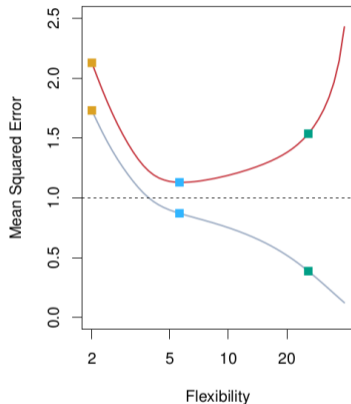
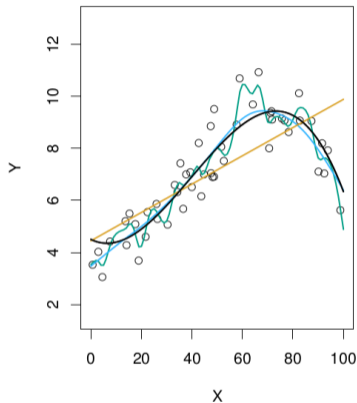
Test set:

The observations

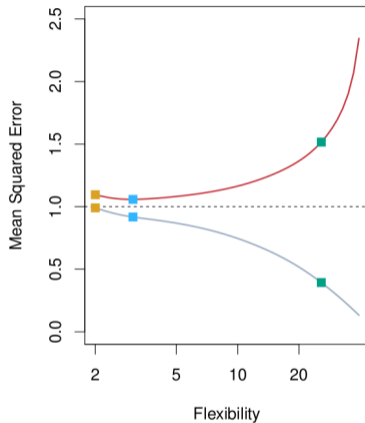
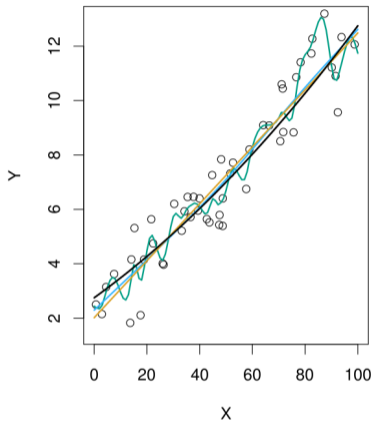
$\{(x'_1, y'_1), \dots, (x'_{n'}, y'_{n'})\}$ used to
compute the average squared
prediction error

$$\frac{1}{n'} \sum_i (y'_i - \hat{f}(x'_i))^2$$

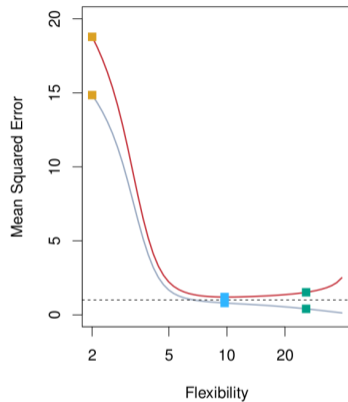
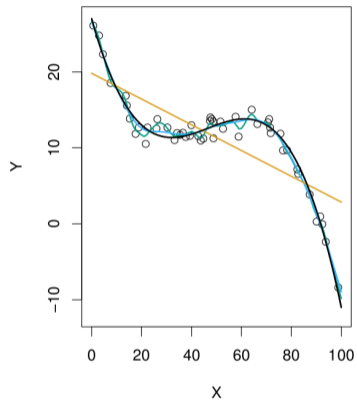
Why not just get the best model for the training data?



A more linear example



A more non-linear example



A simple solution: Train/test split

More on this in Ch 5

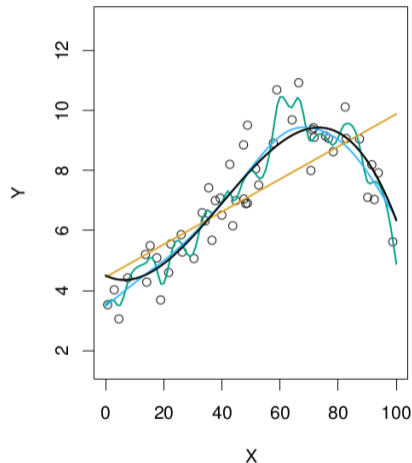
Section 2

Bias-Variance Trade-Off

$$E(y_0 - \hat{f}(x_0))^2 = \text{Var}(\hat{f}(x_0)) + [\text{Bias}(\hat{f}(x_0))]^2 + \text{Var}(\varepsilon)$$

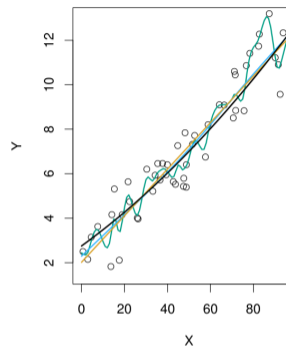
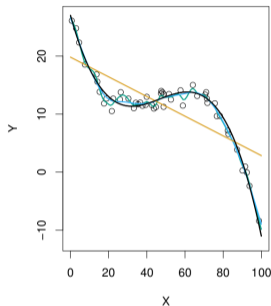
Variance

Variance: the amount by which \hat{f} would change if we estimated it using a different training data set.

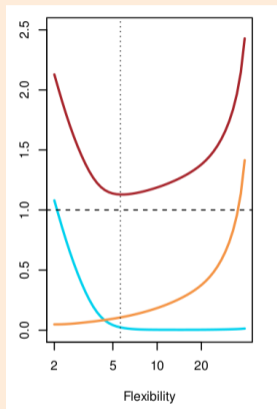
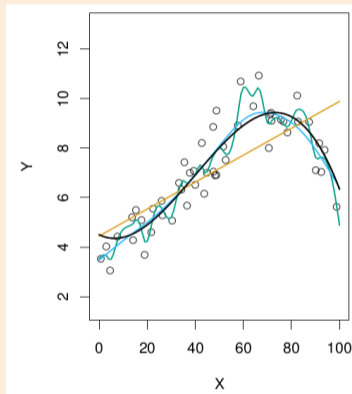


Bias

Bias: the error that is introduced by approximating a (complicated) real-life problem by a much simpler model.



Group work

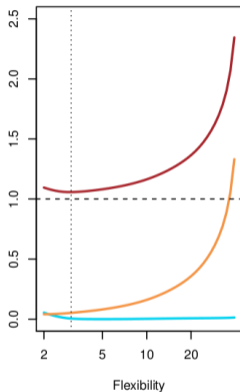
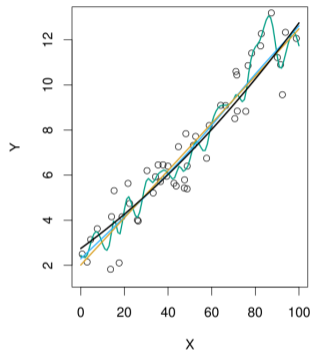


Label the line corresponding to each of the following:

- MSE
- Bias
- Variance of $\hat{f}(x_0)$
- Variance of ε

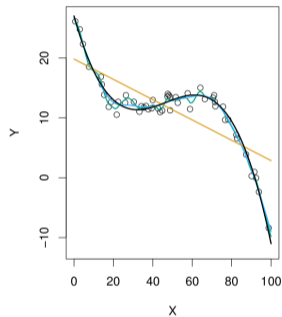
$$E(y_0 - \hat{f}(x_0))^2 = \text{Var}(\hat{f}(x_0)) + [\text{Bias}(\hat{f}(x_0))]^2 + \text{Var}(\varepsilon)$$

Another example

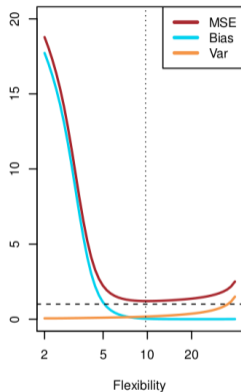


$$E(y_0 - \hat{f}(x_0))^2 = \text{Var}(\hat{f}(x_0)) + [\text{Bias}(\hat{f}(x_0))]^2 + \text{Var}(\varepsilon)$$

Yet another example

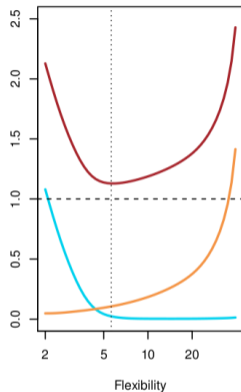


Mean Squared Error



$$E(y_0 - \hat{f}(x_0))^2 = \text{Var}(\hat{f}(x_0)) + [\text{Bias}(\hat{f}(x_0))]^2 + \text{Var}(\varepsilon)$$

Bias-variance trade off



$$E(y_0 - \hat{f}(x_0))^2 = \text{Var}(\hat{f}(x_0)) + [\text{Bias}(\hat{f}(x_0))]^2 + \text{Var}(\varepsilon)$$

Group work: coding

See jupyter notebook

Next time

- Wednesday:
 - ▶ 3.1 Linear Regression
- Sunday
 - ▶ Homework due midnight on Crowdmark

Lec #	Date	Topic	Reading	HW	Pop Quizzes	Notes
1	M 1/13	Intro / Python Review	1			
2	W 1/15	What is statistical learning	2.1		Q1	
3	F 1/17	Assessing Model Accuracy	2.2.1, 2.2.2			
	M 1/20	MLK - No Class				
4	W 1/22	Linear Regression	3.1		Q2	
5	F 1/24	More Linear Regression	3.1	HW #1 Due		
6	M 1/27	Multi-linear Regression	3.2	Sun 1/26		
7	W 1/29	Probably More Linear	3.3		Q3	