Ch 9.2: Support Vector Classifier

Lecture 27 - CMSE 381

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Dept of Computational Mathematics, Science & Engineering

Wed, April 2nd, 2025

Announcements

Last time:

- 9.1 Maximal Margin Classifier

This lecture:

- 9.2 Support Vector Classifier

Announcements:

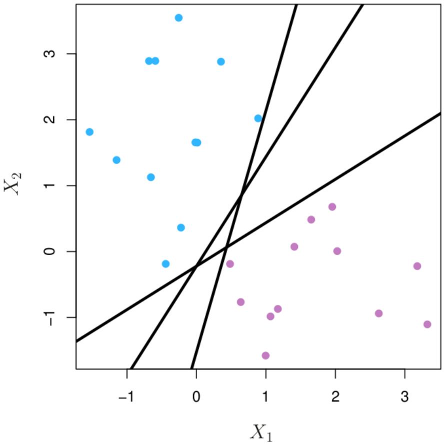
- HW #7 due Sunday 4/6

	M	3/17	**Midterm #2**		Sun 3/16
21	W	3/19	Polynomial & Step Functions	7.1-7.2	
22	F	3/21	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	M	3/24	Regression Splines	7.4	
24	W	3/26	Decision Trees	8.1	HW #6 Due Wed 3/26
25	F	3/28	Random Forests	8.2.1, 8.2.2	HW #7 Due Sun 3/30
26	M	3/31	Maximal Margin Classifier	9.1	
27	W	4/2	SVC	9.2	
28	F	4/4	SVM	9.3, 9.4	HW #8 Due Sun 4/6
29	M	4/7	Single Layer NN	10.1	
30	W	4/9	Multi Layer NN	10.2	
31	F	4/11	CNN	10.3	HW #9 Due Sun 4/13
32	M	4/14	Unsupervised learning / clustering	12.1, 12.4	
33	W	4/16	Virtual: Project Office Hours		
	F	4/18	***Review***		
	M	4/21	**Midterm #3**		
	W	4/23			
	F	4/25			**Project Due**

Section 1

Last time

Separating Hyperplane



Require that for every data point:

$$\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} > 0 \text{ if } y_i = 1$$
$$\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} < 0 \text{ if } y_i = -1$$

Equivalently

Require that for every data point

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) > 0$$

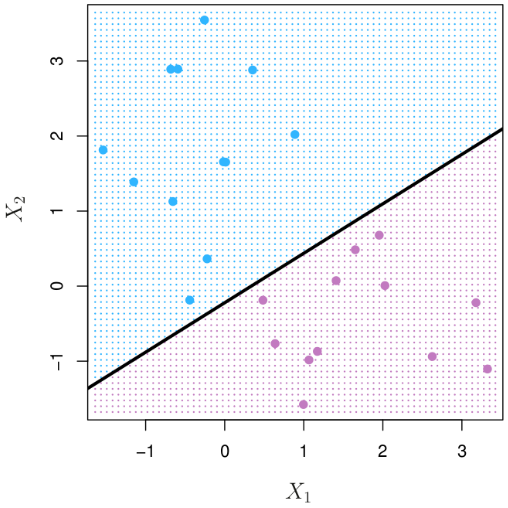
Separating hyperplane becomes a classifier

If you have a separating hyperplane:

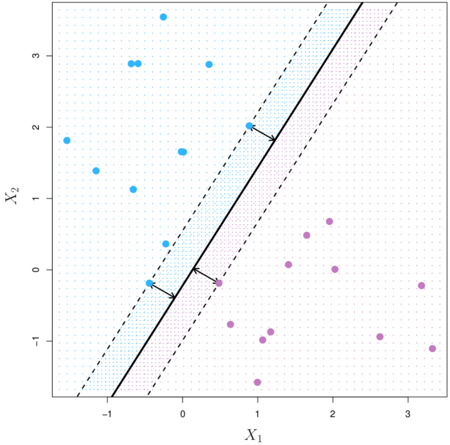
- Check

$$f(x^*) = \beta_0 + \beta_1 x_1^* + \beta_2 x_2^* + \cdots + \beta_p x_p^*$$

- If positive, assign $\hat{y} = 1$
- If negative, assign $\hat{y} = -1$



Maximal margin classifier



- For a hyperplane, the *margin* is the smallest distance from any data point to the hyperplane.
- Observations that are closest are called *support vectors*.
- The *maximal margin hyperplane* is the hyperplane with the largest margin
- The classifier built from this hyperplane is the *maximal margin classifier*.

Test your understanding: PollEv

Mathematical Formulation

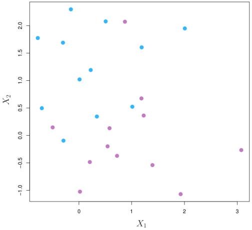
$$\underset{\beta_0,\beta_1,\ldots,\beta_p,M}{\text{maximize}} \; M$$

$$\text{subject to} \; \sum_{j=1}^{p} \beta_j^2 = 1,$$

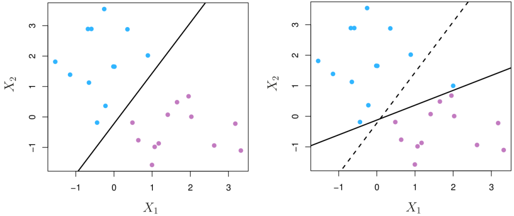
$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M \;\; \forall \; i = 1, \ldots, n$$

Problems

Might be no separating hyperplane



Sensitivity to new points

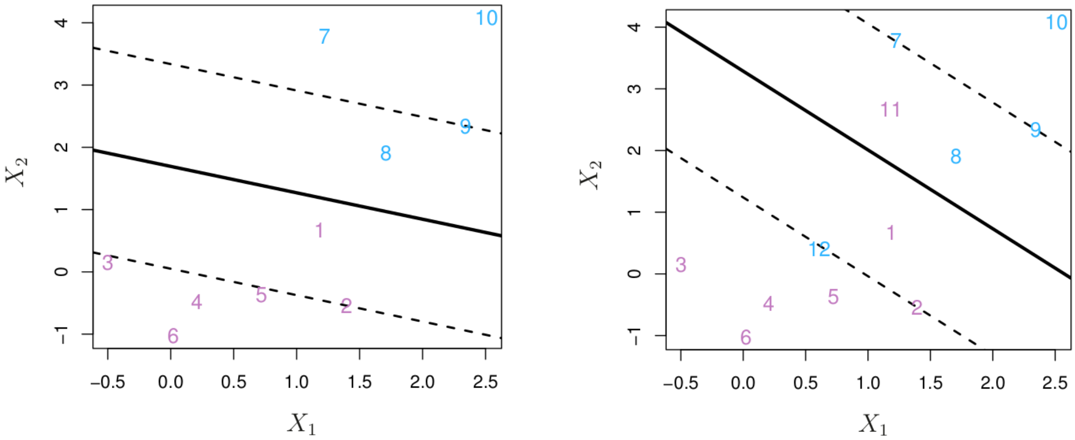


Section 2

Support Vector Classifier

Basic idea

Soft margin



Mathematical Formulation of SVC

$$\underset{\beta_0,\beta_1,\ldots,\beta_p,\epsilon_1,\ldots,\epsilon_n,\, M}{\text{maximize}} \quad M$$

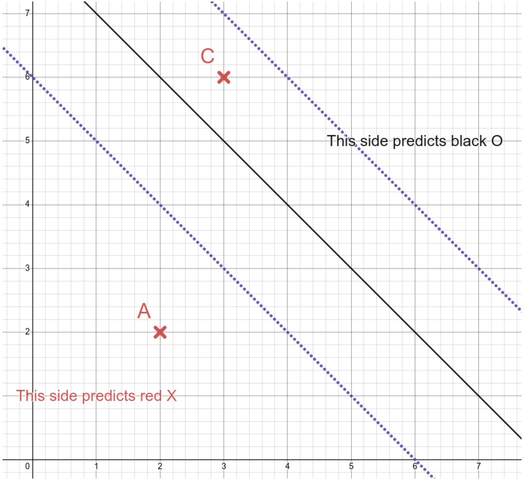
$$\text{subject to } \sum_{j=1}^{p} \beta_j^2 = 1,$$

$$y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M(1 - \epsilon_i),$$

$$\epsilon_i \geq 0, \quad \sum_{i=1}^{n} \epsilon_i \leq C,$$

Find positive ε's that will satisfy this

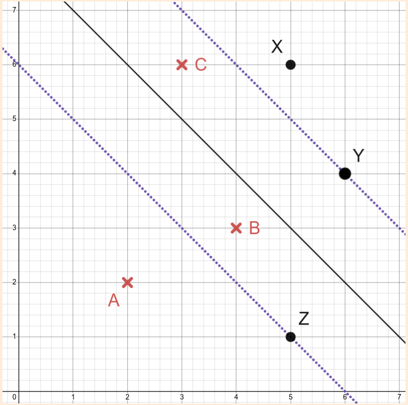
$$\text{Fix } M = \sqrt{2} \qquad y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M(1 - \varepsilon_i)$$



What is ε?

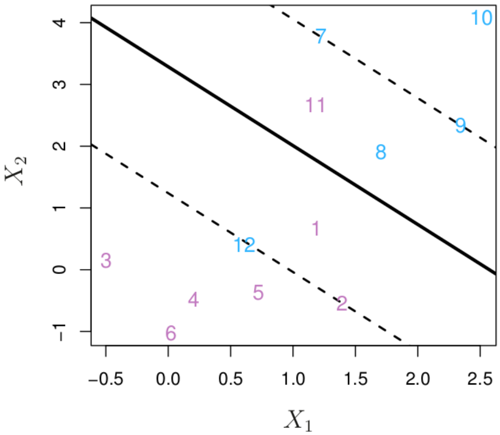
Fix $M = \sqrt{2}$ $\quad y_i(\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip}) \geq M(1 - \varepsilon_i)$

Fill in the table so that the inequality is satisfied.



Point	Left Side	ε_i	$M(1-\varepsilon_i)$
A	$2\sqrt{2}$	0	$\sqrt{2}$
B	$\frac{\sqrt{2}}{2}$		
C	$-\frac{\sqrt{2}}{2}$	1.5	$-\frac{\sqrt{2}}{2}$
X	$\frac{3\sqrt{2}}{2}$		
Y	$\sqrt{2}$		
Z	$-\sqrt{2}$		

What is ε?



What is C?

$$\underset{\beta_0,\beta_1,\ldots,\beta_p,\epsilon_1,\ldots,\epsilon_n,\,M}{\text{maximize}} \quad M$$

$$\text{subject to} \quad \sum_{j=1}^{p} \beta_j^2 = 1,$$

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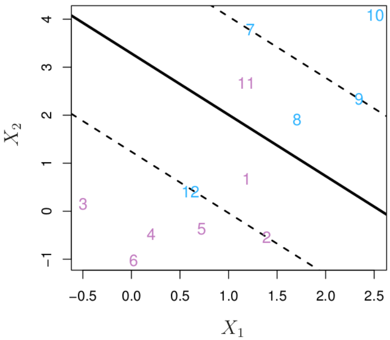
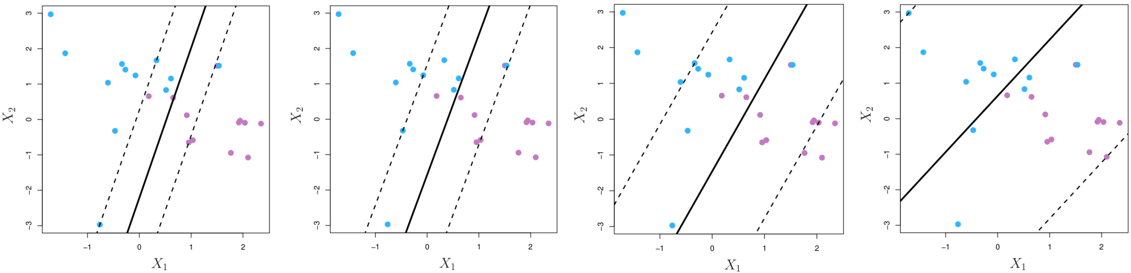
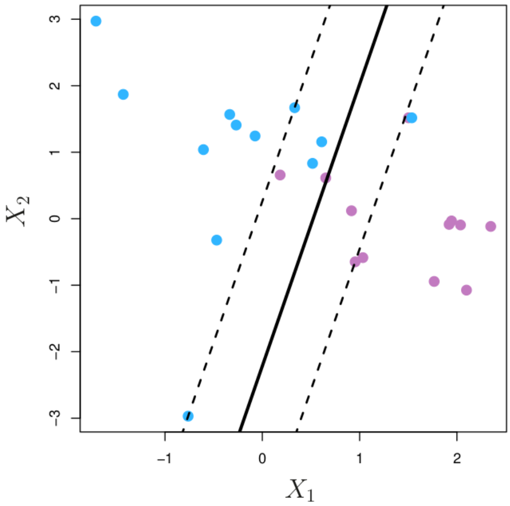


Examples messing with C



Increasing C $\rightarrow$

What affects the hyperplane?



Coding

Maximal Margin Classifier

$$\underset{\beta_0,\beta_1,\ldots,\beta_p,M}{\text{maximize}} \; M$$

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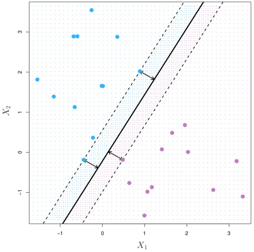

Support Vector Classifier

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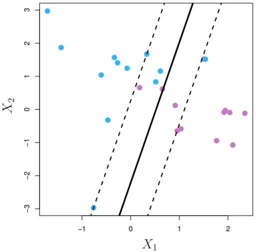

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Q of the Day: what is ϵ in $\cdots > M(1 - \epsilon_i)$ called?