Ch 3.1: Linear Regression
Lecture 4 - CMSE 381

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Announcements

Last time:

- 2.2 Assessing Model Accuracy

Announcements:

- Office Hours

Covered in this lecture

- Least squares coefficient estimates for linear regression
- Residual sum of squares (RSS)

Section 1

Simple Linear Regression

Setup

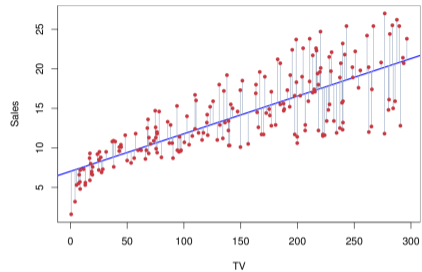- Predict Y on a single predictor variable X

$$Y \approx \beta_0 + \beta_1 X$$

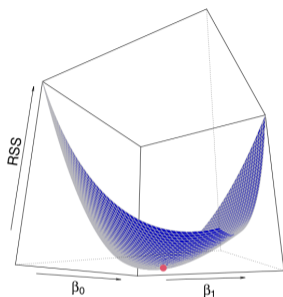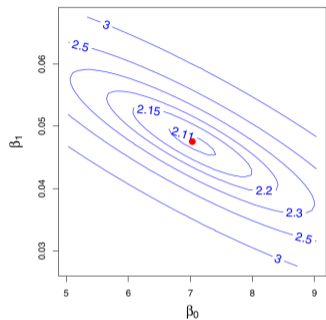- "$\approx$" .... "is approximately modeled as"

Example

		TV	Radio	Newspaper	Sales
2	1	230.1	37.8	69.2	22.1
3	2	44.5	39.3	45.1	10.4
4	3	17.2	45.9	69.3	9.3
5	4	151.5	41.3	58.5	18.5
6	5	180.8	10.8	58.4	12.9
7	6	8.7	48.9	75	7.2
8	7	57.5	32.8	23.5	11.8
9	8	120.2	19.6	11.6	13.2
10	9	8.6	2.1	1	4.8
11	10	199.8	2.6	21.2	10.6
12	11	66.1	5.8	24.2	8.6

Least squares criterion: Setup

How do we estimate the coefficients?

Least squares criterion: RSS

Residual sum of squares RSS is

$$RSS = e_1^2 + \cdots + e_n^2$$
$$= \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\text{sales} \approx \beta_0 + \beta_1 \text{TV}$$

Least squares criterion

Find β_0 and β_1 that minimize the RSS.

Least squares coefficient estimates

$$\min_{\beta_0, \beta_1} \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\frac{\partial RSS}{\partial \beta_0} = -2 \sum_i (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\frac{\partial RSS}{\partial \beta_1} = -2 \sum_i x_i (y_i - \beta_0 - \beta_1 x_i) = 0$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^n (x_i - \overline{x})^2}$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}$$

Coding group work

Next time

Lec #	Date		Topic	Reading	HW	Pop Quizzes	Notes
1	M	1/13	Intro / Python Review	1			
2	W	1/15	What is statistical learning	2.1		Q1	
3	F	1/17	Assessing Model Accuracy	2.2.1, 2.2.2			
	M	1/20	MLK - No Class				
4	W	1/22	Linear Regression	3.1		Q2	
5	F	1/24	More Linear Regression	3.1	HW #1 Due Sun 1/26		
6	M	1/27	Multi-linear Regression	3.2			
7	W	1/29	Probably More Linear Regression	3.3		Q3	

Announcements

- Homework 1
  - ▶ Due Sun, Jan 26