Ch 5.1.1-2: Leave One Out Cross-validation

Lecture 12 - CMSE 381

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Announcements

Last time:

- Exam

Announcements:

- Exam 1 grades.... hopefully soon
- HW #4 will be posted later next week.
  - Changed Deadline! Due Wednesday Mar 2nd.

	W	2/12	**Midterm #1**		
12	F	2/14	Leave one out CV	5.1.1, 5.1.2	
13	M	2/17	k-fold CV	5.1.3	
14	W	2/19	More k-fold CV	5.1.4-5	
15	F	2/21	k-fold CV for classification	5.1.5	
16	M	2/24	Subset selection	6.1	
17	W	2/26	Shrinkage: Ridge	6.2.1	
18	F	2/28	Shrinkage: Lasso	6.2.2	HW #4 Due Sun 3/2
	M	3/3	Spring Break		
	W	3/5	Spring Break		
	F	3/7	Spring Break		
19	M	3/10	PCA	6.3	
20	W	3/12	PCR	6.3	
	F	3/14	***Review***		HW #5 Due Sun 3/16
	M	3/17	**Midterm #2**		

Covered in this lecture

- Validation Set
- LOO CV

Section 1

Validation set

What's the problem?

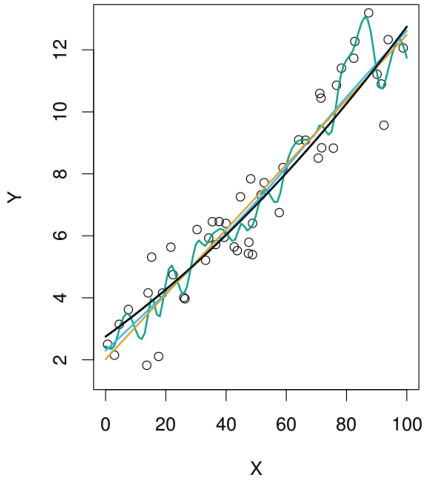
- How well is my ML method doing? *Model Assessment*
- Which method is best for our data?
- How many features should I use? Which ones? *Model selection*
- What is the uncertainty in the learned parameters?

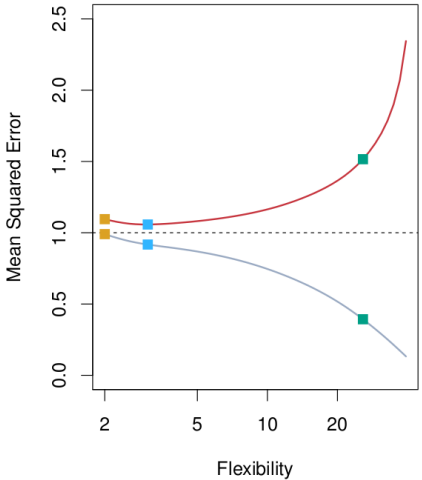
Training Error vs Testing Error

Training Error

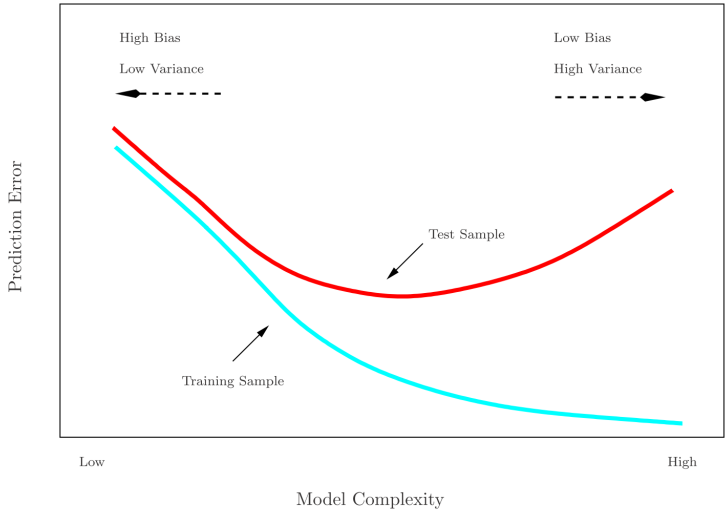
Testing Error

Throw-back

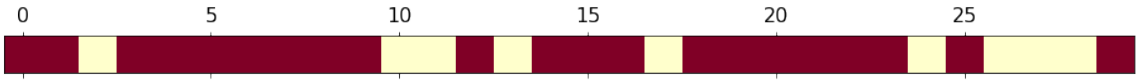

Y
X
2
4
6
8
10
12
0
20
40
60
80
100


Mean Squared Error
Flexibility
0.0
0.5
1.0
1.5
2.0
2.5
2
5
10
20

Model tradeoffs

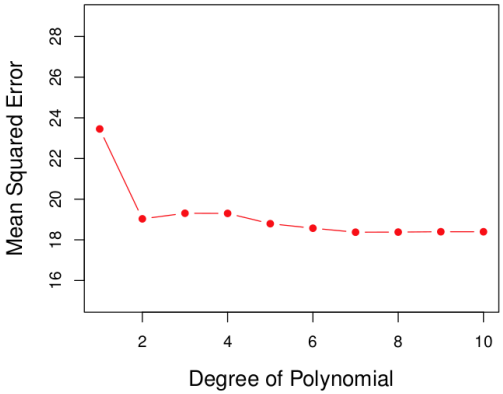


Validation set approach



- Divide randomly into two parts:
  - Training set
  - Validation/Hold-out/Testing set
- Fit model on training set
- Use fitted model to predict response for observations in the test set
- Evaluate quality (e.g. MSE)

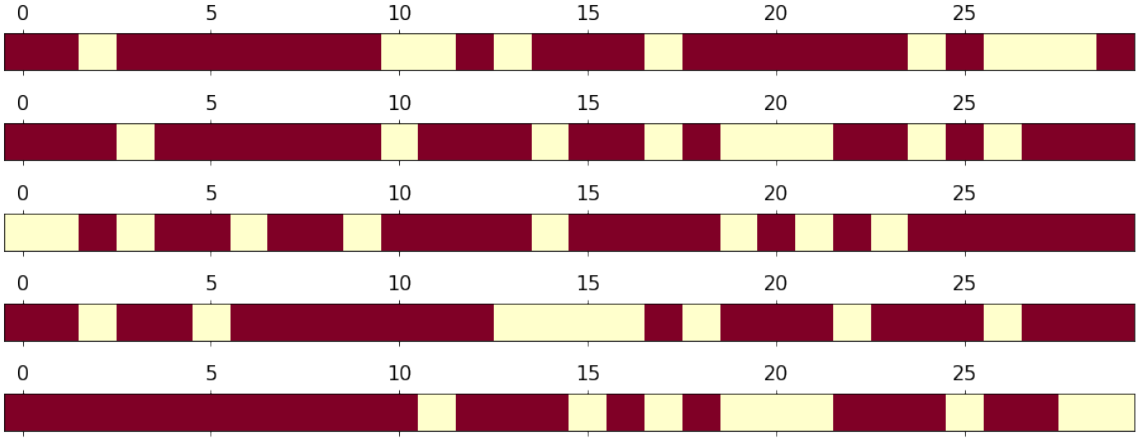
Example with the auto data



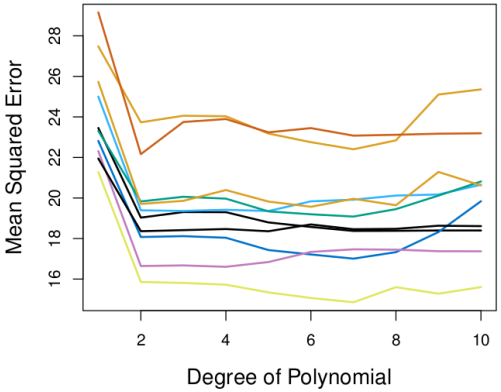
Predicting mpg using horsepower:

$$\texttt{mpg} = \beta_0 + \beta_1 \texttt{hp} + \beta_2 \texttt{hp}^2 + \cdots + \beta_p \texttt{hp}^p$$

Rinse and repeat



Again example with auto data

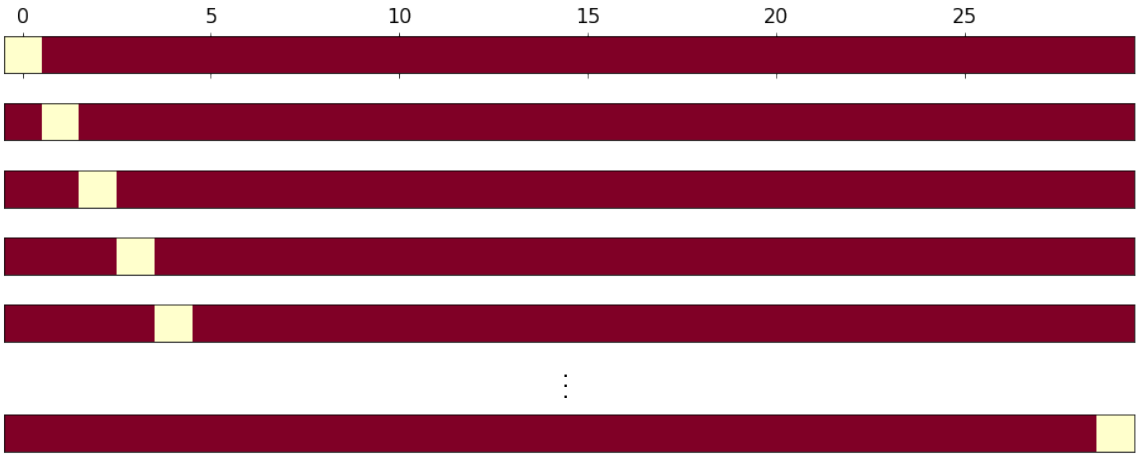


Coding example in jupyter notebook

Section 2

Leave-One-Out Cross-Validation (LOOCV)

The idea



The idea in mathy words

- Remove (x_1, y_1) for testing.
- Train the model on $n-1$ points: $\{(x_2, y_2), \cdots, (x_n, y_n)\}$
- Calculate $\mathrm{MSE}_1 = (y_1 - \hat{y}_1)^2$

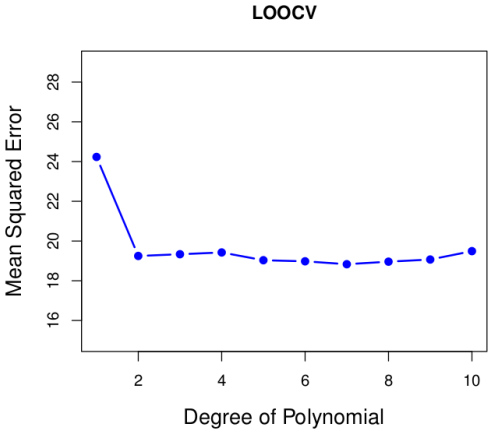
- Remove (x_2, y_2) for testing.
- Train the model on $n-1$ points: $\{(x_1, y_1), (x_3, y_3), \cdots, (x_n, y_n)\}$
- Calculate $\mathrm{MSE}_2 = (y_2 - \hat{y}_2)^2$

- Rinse and repeat

Return the score:

$$CV_{(n)} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{MSE}_i$$

Again example with auto data



Do the LOOCV coding section

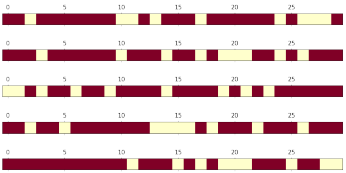
LOOCV Pros and Cons

Advantages:

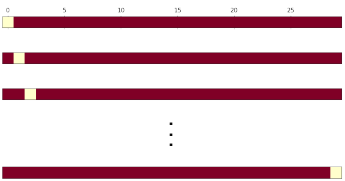
Disadvantages:

TL;DR

Validation set



LOO-CV



LOO-CV Score

$$CV_{(n)} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{MSE}_i$$

Next time

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