Ch 2.2.3: Intro to classification

Lecture 9 - CMSE 381

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Announcements

Lec #	Date		Topic	Reading	HW	Pop Quizzes	Notes
1	M	1/13	Intro / Python Review	1			
2	W	1/15	What is statistical learning	2.1		Q1	
3	F	1/17	Assessing Model Accuracy	2.2.1, 2.2.2			
	M	1/20	MLK - No Class				
4	W	1/22	Linear Regression	3.1		Q2	
5	F	1/24	More Linear Regression	3.1	HW #1 Due Sun 1/26		
6	M	1/27	Multi-linear Regression	3.2			
7	W	1/29	Probably More Linear Regression	3.3		Q3	
8	F	1/31	Last of the Linear Regression		HW #2 Due Sun 2/1		
9	M	2/3	Intro to classification, Bayes classifier, KNN classifier	2.2.3			
10	W	2/5	Logistic Regression	4.1, 4.2, 4.3.1-3		Q4	
11	F	2/7	Multiple Logistic Regression / Multinomial Logistic Regression	4.3.4-5	HW #3 Due Sun 2/9		
	M	2/10	***Project Day & Review***				
	W	2/12	**Midterm #1**				

Last Time:

- Finished Linear Regression

Announcements:

- Homework #3 Due Sunday Feb 9
- Next Monday - Review day
  - Nothing prepped
  - Bring your questions
- Wednesday 2/12 - Exam #1
  - Bring 8.5x11 sheet of paper
  - Handwritten both sides
  - Anything you want on it, but must be your work
  - You will turn it in

Covered in this lecture

- Ch 2.2.3
- Error rate (classification)
- Bayes Classifier
- K-NN classification

Section 1

Classification Overview

What is classification

Classification: When the response variable is qualitative

- Given feature vector X and qualitative response Y in the set S, the goal is to find a function (classifier) $C(X)$ taking X as input and predicting its value for Y.
- We are more interested in estimating the probabilities that X belongs to each category

Some examples

- Predict whether a COVID19 vaccine will work on a patient given patient's age
- An online banking service wants to determine whether a transaction being performed is fraudulent on the basis of the user's IP address, past transactions, etc.

Section 2

Ch 2.2.3: Classification

Error rate

- Training data: $\{(x_1, y_1), \cdots, (x_n, y_n)\}$ with y_i qualitative
- Estimate $\hat{y} = \hat{f}(x)$
- Indicator variable

Training error rate:

$$\frac{1}{n}\sum_{i=1}^{n} \mathrm{I}(y_i \neq \hat{y}_i)$$

Test error rate:

$$\mathrm{Ave}(\mathrm{I}(y_0 \neq \hat{y}_0))$$

Best ever classifier

We can't have nice things

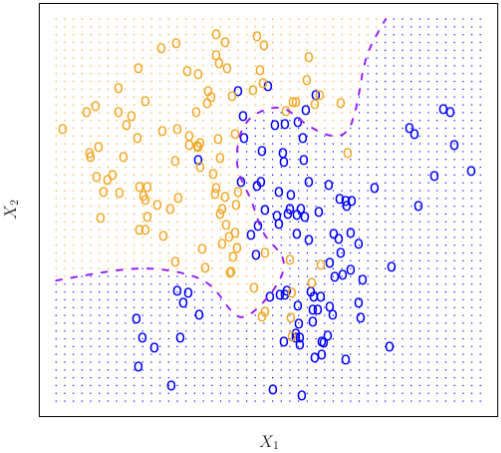
Bayes Classifier:
Give every observation the highest probability class given its predictor variables

$$\Pr(Y = j \mid X = x_0)$$

An example

- Survey students for amount of programming experience, and current GPA
- Try to predict if they will pass CMSE 381.
- If we have a survey of all students that could ever exist, we can determine the probability of failure given combo of those features.

Bayes decision boundary



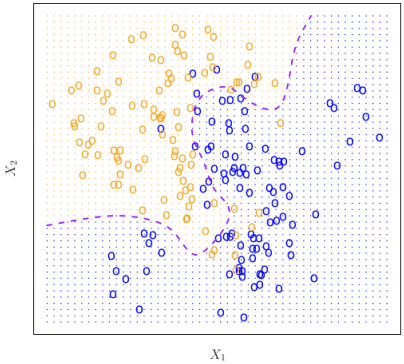
Bayes error rate

- Error at $X = x_0$

$$1 - \max_j \Pr(Y = j \mid X = x_0)$$

- Overall Bayes error:

$$1 - E\left(\max_j \Pr(Y = j \mid X = x_0)\right)$$

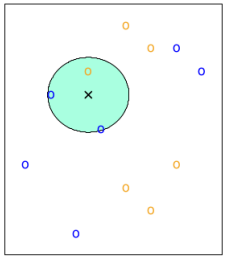


The game

Section 3

K-Nearest Neighbors Classifier

K-Nearest Neighbors

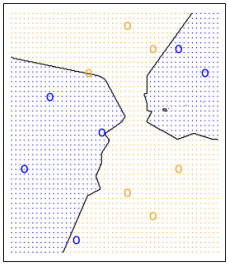


$K = 3$

- Fix K positive integer
- $N(x)$ = the set of K closest neighbors to x
- Estimate conditional proability

$$\Pr(Y = j \mid X = x_0) = \frac{1}{K} \sum_{i \in N(x_0)} \mathrm{I}(y_i = j)$$

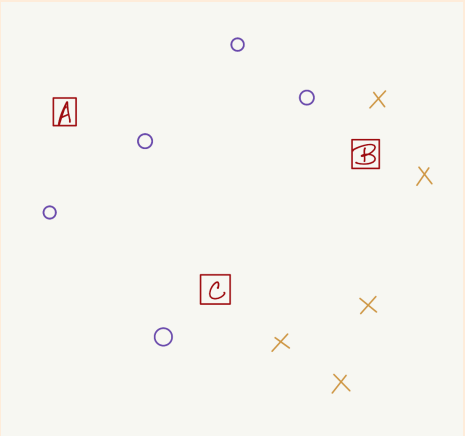
- Pick j with highest value



Black line: KNN decision boundary

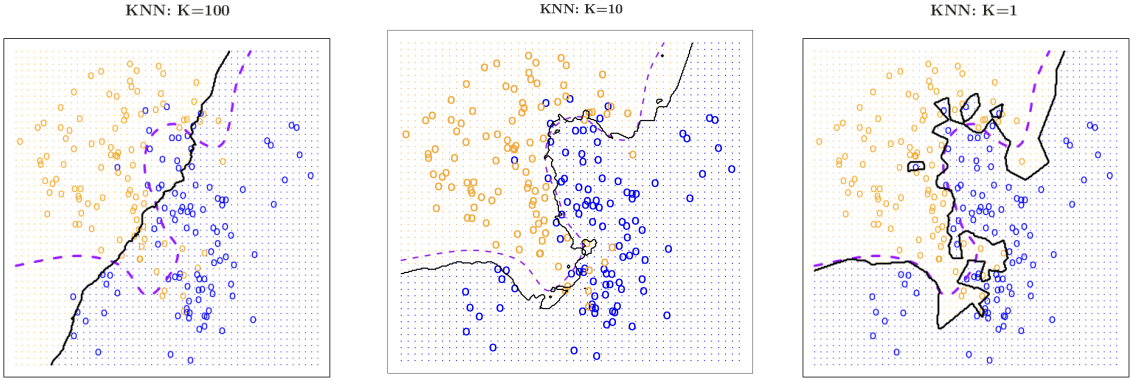
Example

Here label is shown by O vs X. What are the knn predictions for points A, B and C for $k = 1$ or $k = 3$?

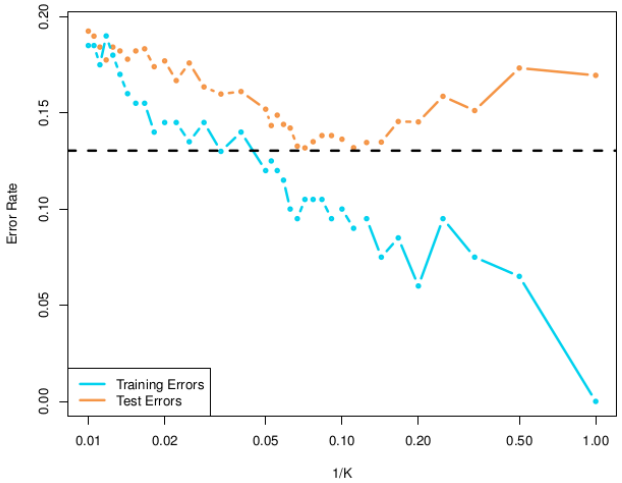


Point	$k = 1$ Prediction	$k = 3$ Prediction
A		
B		
C		

Tradeoff



More on tradeoff



Jupyter notebook

Next time

- Weds 2/5
  - Logistic Regression

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Announcements

- Homework 3
  - Due Sun, Feb 9