Ch 3.2: Multiple Linear Regression

Lecture 6 - CMSE 381

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Announcements

Last time:

- 3.1 Linear regression

Announcements:

- Homework #2 Due Sunday, Feb 1, on Crowdmark

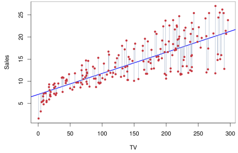
Covered in this lecture

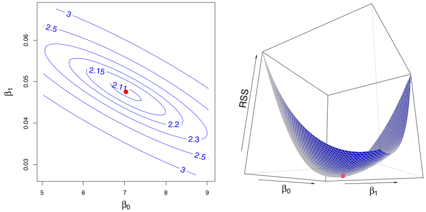
- Multiple linear regression
- Hypothesis test in that case
- Forward and Backward Selection

Section 1

Review from last time

Linear Regression with One Variable





- Predict Y on a single variable X

$$Y \approx \beta_0 + \beta_1 X$$

- Find good guesses for $\hat{\beta}_0$, $\hat{\beta}_1$.
- $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$
- $e_i = y_i - \hat{y}_i$ is the ith residual
- RSS $= \sum_i e_i^2$

- RSS is minimized at *least squares coefficient estimates*

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^n (x_i - \overline{x})^2}$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}$$

Evaluating the model

- Linear regression is unbiased
- Variance of linear regression estimates:

$$\mathrm{SE}(\hat{\beta}_0) = \sigma^2 \left[\frac{1}{n} + \frac{\overline{x}^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2} \right]$$

$$\mathrm{SE}(\hat{\beta}_1)^2 = \frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \overline{x})^2}$$

  where $\sigma^2 = \mathrm{Var}(\varepsilon)$
- Estimate σ: $\hat{\sigma}^2 = \frac{RSS}{n-2}$

- The 95% confidence interval for β_1 approximately takes the form

$$\hat{\beta}_1 \pm 2 \cdot \mathrm{SE}(\hat{\beta}_1)$$

- Hypothesis test:
  H_0: $\beta_1 = 0$
  H_a: $\beta_1 \neq 0$
  - Test statistic $t = \frac{\hat{\beta}_1 - 0}{\mathrm{SE}(\hat{\beta}_1)}$

Assessing the accuracy of the model

Residual standard error (RSE):

$$RSE = \sqrt{\frac{1}{n-2} RSS}$$

R squared:

$$R^2 = \frac{TSS - RSS}{TSS} = 1 - \frac{RSS}{TSS}$$

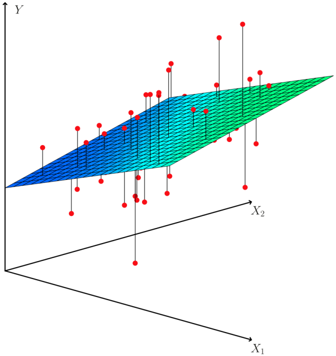
$$TSS = \sum_i (y_i - \overline{y})^2$$

Section 2

Multiple Linear Regression

Setup

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots \beta_p X_p + \varepsilon$$



Estimation and Prediction

Given estimates $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \cdots, \hat{\beta}_p$, prediction is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \cdots + \hat{\beta}_p x_p$$

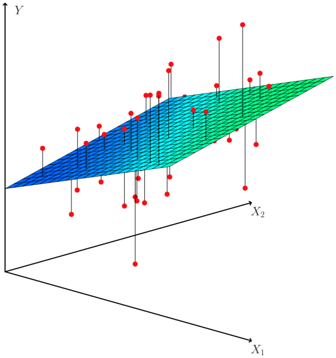
Minimize the sum of squares

$$\begin{aligned} RSS &= \sum_i (y_i - \hat{y}_i)^2 \\ &= \sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_1 - \cdots - \hat{\beta}_p x_p) \end{aligned}$$

Coefficients are closed form but UGLY

Advertising data set example

$$\texttt{Sales} = \beta_0 + \beta_1 \cdot \texttt{TV} + \beta_2 \cdot \texttt{radio} + \beta_3 \cdot \texttt{newspaper}$$



	Coefficient
Intercept	2.939
TV	0.046
radio	0.189
newspaper	−0.001

Interpretation of coefficients

$$\texttt{Sales} = \beta_0 + \beta_1 \cdot \texttt{TV} + \beta_2 \cdot \texttt{radio} + \beta_3 \cdot \texttt{newspaper}$$

	Coefficient
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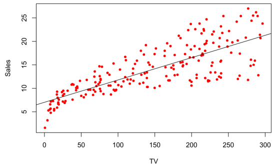
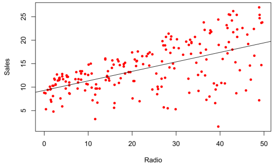
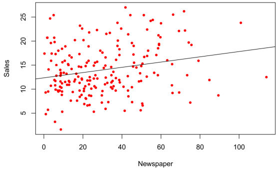


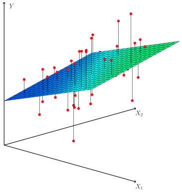
	Coefficient
Intercept	7.0325
TV	0.0475



	Coefficient
Intercept	9.312
radio	0.203



	Coefficient
Intercept	12.351
newspaper	0.055



	Coefficient
Intercept	2.939
TV	0.046
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newspaper	−0.001

Correlation matrix

	TV	radio	newspaper	sales
TV	1.0000	0.0548	0.0567	0.7822
radio		1.0000	0.3541	0.5762
newspaper			1.0000	0.2283
sales				1.0000

Coding group work

Run the section titled "Multiple Linear Regression"

Section 3

Ch 3.2.2: Questions to ask of your regression

Question 1

Is at least one of the predictors $X_1, \cdots, X_p$ useful in predicting the response?

Q1: Hypothesis test

H_0 : $\beta_1 = \beta_2 = \cdots = \beta_p = 0$

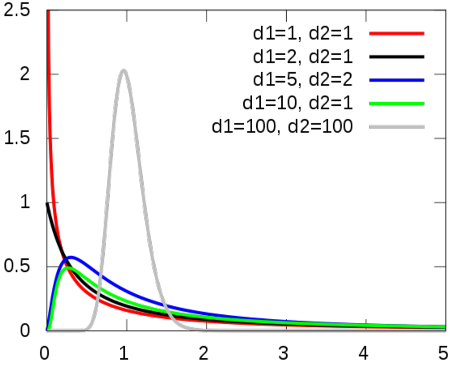
H_a : At least one β_j is non-zero

F-statistic:

$$F = \frac{(TSS - RSS)/p}{RSS/(n-p-1)} \sim F_{p,n-p-1}$$

The F-statistic for the hypothesis test

$$F = \frac{(TSS - RSS)/p}{RSS/(n-p-1)} \sim F_{p,n-p-1}$$



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Do Q1 section in jupyter notebook

Q2

Do all the predictors help to explain Y, or is only a subset of the predictors useful?

Q2: A first idea

Great, you know at least one variable is important, so which is it?....

Do Q2 section in jupyter notebook

Why is this a bad idea?

Next time

- Weds 1/29
  - More Linear Regression

Lec #	Date		Topic	Reading	HW	Pop Quizzes	Notes
1	M	1/13	Intro / Python Review	1			
2	W	1/15	What is statistical learning	2.1		Q1	
3	F	1/17	Assessing Model Accuracy	2.2.1, 2.2.2			
	M	1/20	MLK - No Class				
4	W	1/22	Linear Regression	3.1		Q2	
5	F	1/24	More Linear Regression	3.1	HW #1 Due Sun 1/26		
6	M	1/27	Multi-linear Regression	3.2			
7	W	1/29	Probably More Linear Regression	3.3		Q3	
8	F	1/31	Last of the Linear Regression		HW #2 Due Sun 2/1		
9	M	2/3	Intro to classification, Bayes classifier, KNN classifier	2.2.3			
10	W	2/5	Logistic Regression	4.1, 4.2, 4.3.1-3		Q4	
11	F	2/7	Multiple Logistic Regression / Multinomial Logistic Regression	4.3.4-5	HW #3 Due Sun 2/9		
	M	2/10	***Project Day & Review***				
	W	2/12	**Midterm #1**				

Announcements

- Homework 2
  - Due Sun, Feb 1st