

Ch 6.3: PCR

Lecture 20 - CMSE 381

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Dept of Computational Mathematics, Science & Engineering

Wed, March 12, 2025

Announcements

Last time:

- PCA

This lecture:

- PCR

Announcements:

- Exam #2 on Monday!
 - ▶ Bring 8.5x11 sheet of paper
 - ▶ Handwritten both sides
 - ▶ Anything you want on it, but must be your work
 - ▶ You will turn it in
 - ▶ Non-internet calculator
 - ▶ questions about project

	W	2/12	Midterm #1		
12	F	2/14	Leave one out CV	5.1.1, 5.1.2	
13	M	2/17	k-fold CV	5.1.3	
14	W	2/19	More k-fold CV	5.1.4-5	
15	F	2/21	k-fold CV for classification	5.1.5	
16	M	2/24	Subset selection	6.1	
17	W	2/26	Shrinkage: Ridge	6.2.1	
18	F	2/28	Shrinkage: Lasso	6.2.2	HW #4 Due Sun 3/2
	M	3/3	Spring Break		
	W	3/5	Spring Break		
	F	3/7	Spring Break		
19	M	3/10	PCA	6.3	
20	W	3/12	PCR	6.3	
	F	3/14	Review		HW #5 Due Sun 3/16
	M	3/17	Midterm #2		

Burning questions before midterm
#2 of CMSE 381 (Spring 2025)



Section 1

Previously...

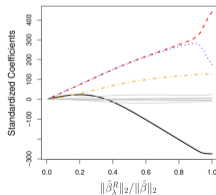
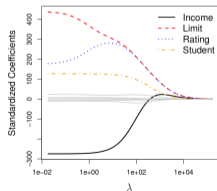
Shrinkage

Find β to minimize

$$RSS = \sum_{i=1}^n \left(y_i - \beta_0 - \sum_{j=1}^p \beta_j x_{ij} \right)^2$$

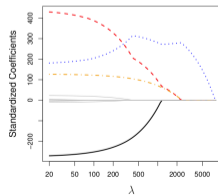
subject to:

Least Squares:
No constraints



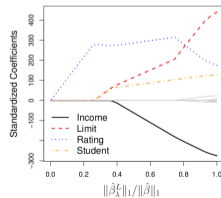
Ridge:

$$\sum_{j=1}^p \beta_j^2 \leq s$$



The Lasso:

$$\sum_{j=1}^p |\beta_j| \leq s$$



Linear transformation of predictors

Original Predictors:

$$X_1, \dots, X_p$$

New Predictors:

$$Z_1, \dots, Z_M$$

$$Z_m = \sum_{j=1}^p \varphi_{jm} X_j$$

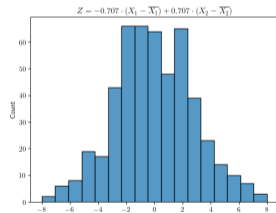
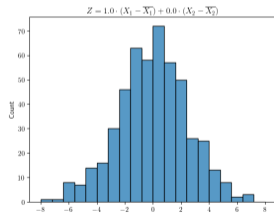
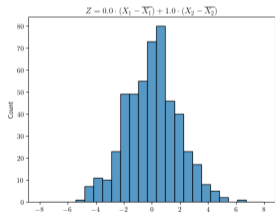
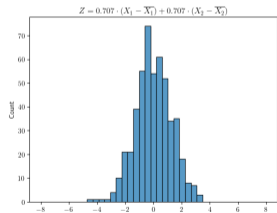
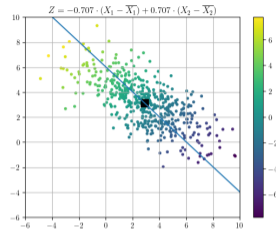
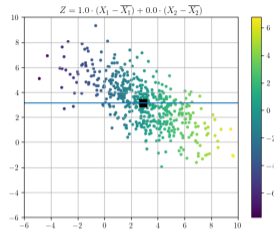
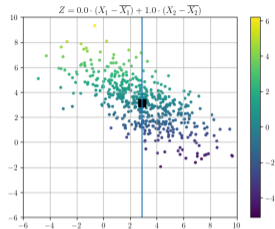
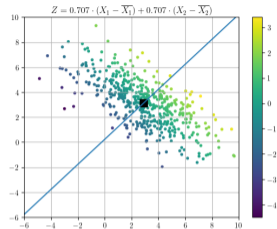
The goal:

- Find good φ 's (PCA)
- Fit regression model on Z_i 's using least squares (PLS)

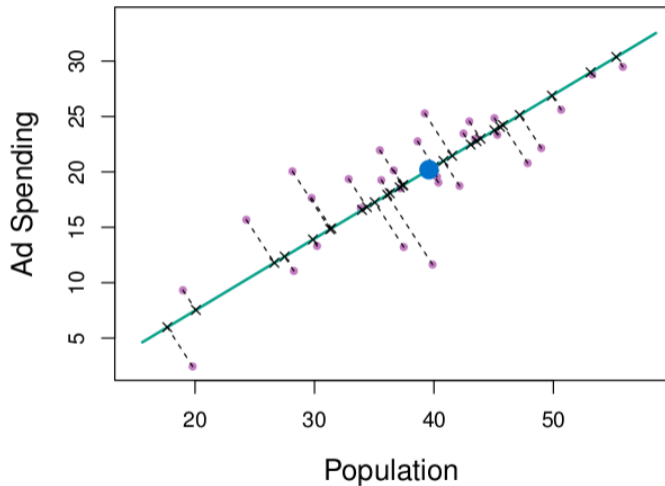
$$y_i = \theta_0 + \sum_{m=1}^M \theta_m z_{im} + \varepsilon_i$$

- Hope that lower dimensions means less overfitting

PCA - First PC

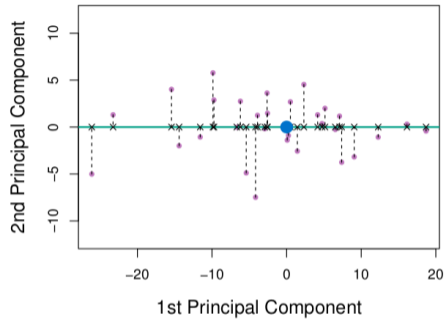
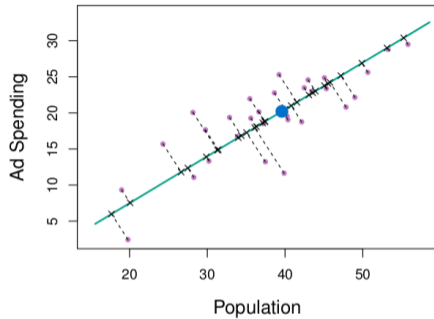


Projection onto first PC



$$Z_1 = 0.839 \cdot (\text{pop} - \overline{\text{pop}}) + 0.544 \cdot (\text{ad} - \overline{\text{ad}})$$

Drawing points in PC space



Section 2

Principal Components Regression

So you've found your PCA coefficients

Now what?

What are we assuming?

Interpretation of PCR coefficients

Original Predictors:

$$X_1, \dots, X_p$$

New Predictors:

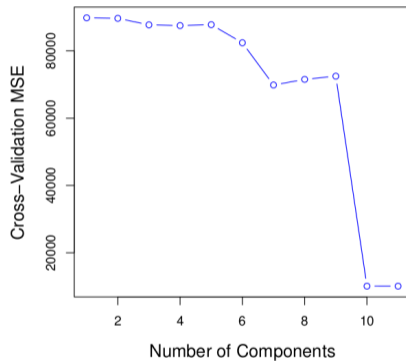
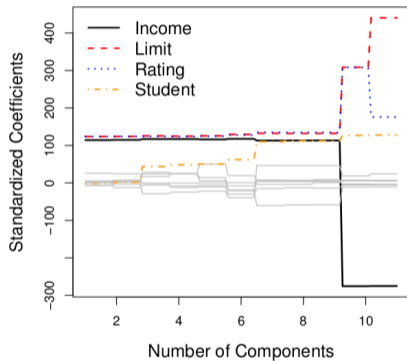
$$Z_1, \dots, Z_M$$

$$Z_m = \sum_{j=1}^p \varphi_{jm} X_j$$

Learned model:

$$y = \theta_0 + \theta_1 Z_1 + \dots + \theta_M Z_M$$

Picking M

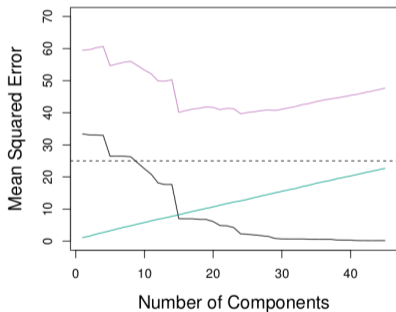


Do PCR with hitters data

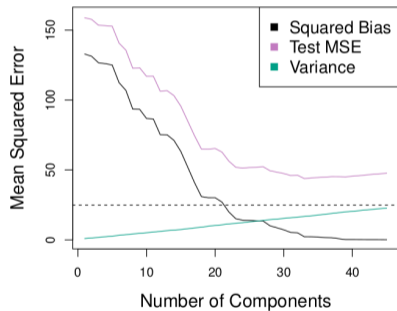
Bias-Variance Trade-off

Example with simulated data: $n = 50$ observations of $p = 45$ predictors

Y is a function of **all predictors**

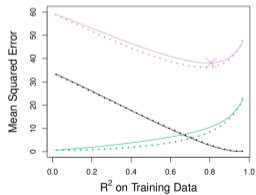
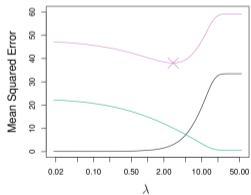
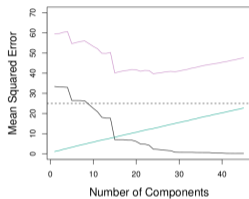


Y is a function of **2 predictors**

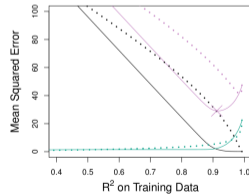
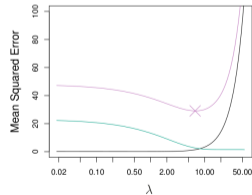
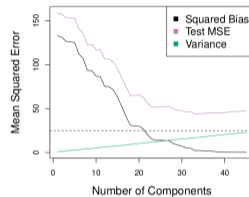


Comparison to results on shrinkage

Y is a function of all predictors



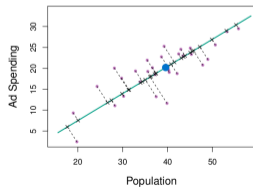
Y is a function of 2 predictors



Properties of PCR

PCR

- Unsupervised dimensionality reduction + linear regression
- Choose component Z_1 in the direction of most variance using only X_j 's information
- Choose Z_2 and beyond by the same method after “getting rid” of info in the directions already explained



Next time

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