Ch 10.1: Neural Nets

Lecture 29 - CMSE 381

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Announcements

Last time:

- SVM

This lecture:

- Feed Forward Neural Nets

Announcements:

- Homework #9 due Sunday, 11/23 11:59pm
- Review class next Friday - start thinking about questions!

21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/26
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	
24	F	10/31	Decision Trees	8.1	HW #6 Due Sun 11/2
25	M	11/3	Random Forests	8.2.1, 8.2.2	
26	W	11/5	Maximal Margin Classifier	9.1	
27	F	11/7	SVC	9.2	HW #7 Due Sun 11/9
28	M	11/10	SVM	9.3, 9.4	
29	W	11/12	Single Layer NN	10.1	
30	F	11/13	Multi Layer NN	10.2	HW #8 Due Sun 11/16
31	M	11/17	CNN	10.3	
32	W	11/19	Unsupervised learning / clustering	12.1, 12.4	
33	F	11/21	***Review***		HW #9 Due Sun 11/23
	M	11/24	**Midterm #3**		
	W	11/26	Virtual: Project Office Hours		
	F	11/28	Thanksgiving		
	M	12/1	Virtual: Project Office Hours		
	W	12/3	Virtual: Project Office Hours		
	F	12/5			**Project Due**
	M	12/8			
	W	12/10			
	F	12/12	**No final exam**		**Honors Project Due**

What will you learn today?

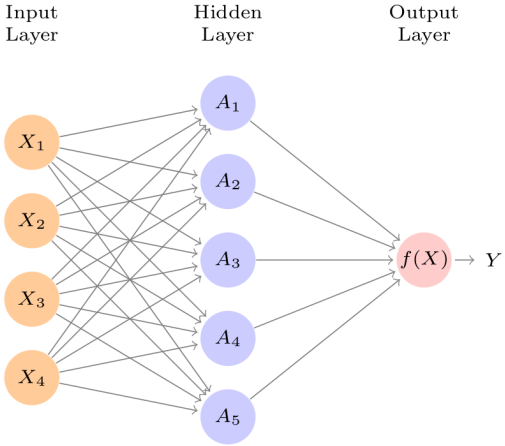
- What is the architecture of simple feed-forward neural network?
  - You should be able to describe what happens at each layer (e.g., input layer, hidden layer, output layer) conceptually and mathematically.
- What is activation? What types of activation functions are there?
  - You should be able to explain it mathematically.
  - Given example inputs and weights from the previous matrix, you should be able to calculate the activation A by hand (with the help of a calculator).
- How does a simple feed-forward neural network produce an output?
  - Given a β matrix and activation from the previous layer, you should be able to calculate the output by hand.
- What do the fitted parameters minimize in a simple feed-forward neural network?

Section 1

Neural Nets

The idea

Feed Forward Neural Network: The cartoon



What is activation? Neuroscience 101.

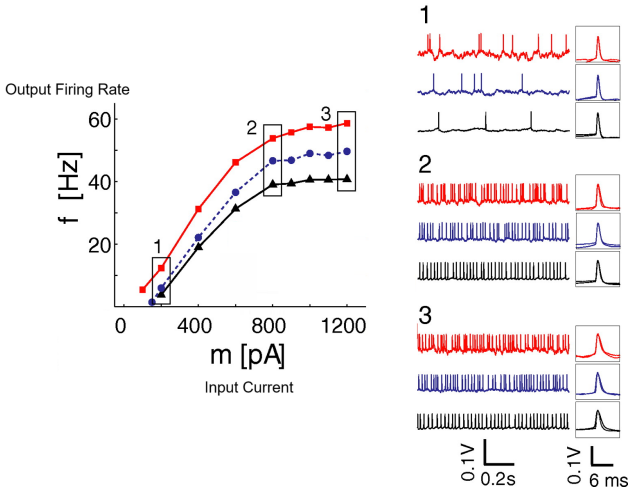
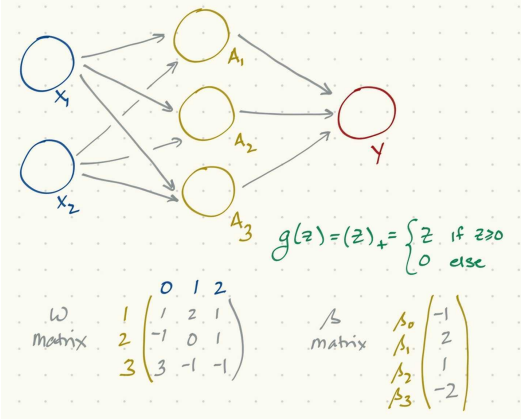


Figure adapted from Fig 1 of Arsiero et al. 2007 (J of Neurosci.)

A very simple example

Computing A_k for $(1, 0)$

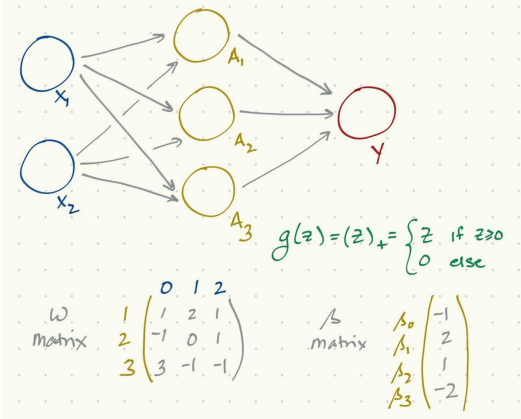
$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$



A very simple example

Computing Y for $(1, 0)$

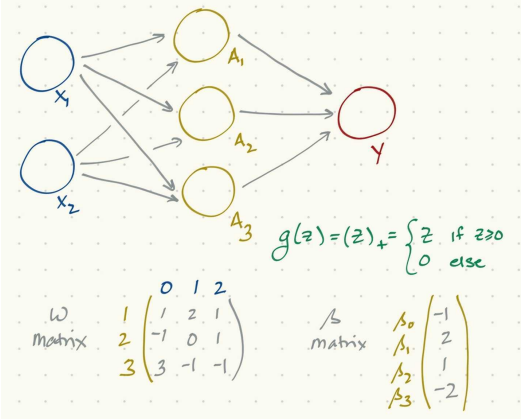
$$f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$



A very simple example

Computing Y for $(0, 1)$

$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j), \qquad f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$



Test your understanding: PollEv!

A different example

- Draw the diagram for a neural net with input data points with $p = 3$ (i.e., (X_1, X_2, X_3)) and two units in the hidden layer.
- Using the ω and β matrices, what is the output predicted Y for the point $(2, 0, 1)$?

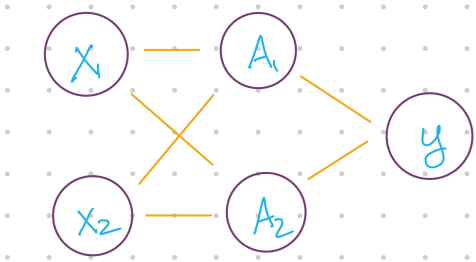
$$\omega = \begin{pmatrix} 1 & 0 & -2 & 2 \\ -3 & 1 & 0 & -1 \end{pmatrix} \qquad \beta = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

- Use the activation function

$$g(z) = (z)_+ = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{else.} \end{cases}$$

Extra space

What if there's no activation function?

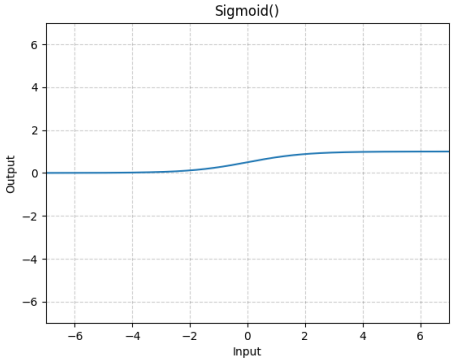


$$\omega = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \qquad \beta = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}$$

Choices for activation function

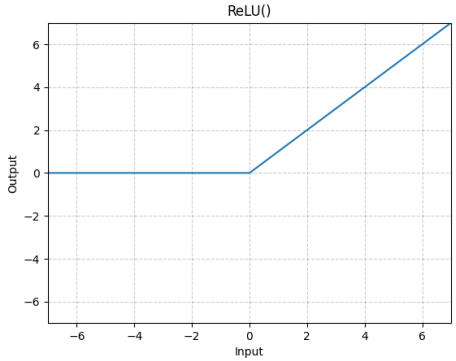
Sigmoid:

$$g(z) = \frac{e^z}{1+e^z} = \frac{1}{1+e^{-z}}$$

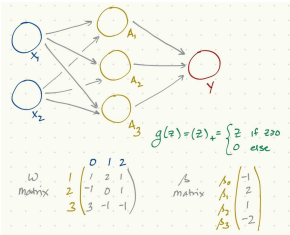


ReLU: Rectified linear unit

$$g(z) = (z)_+ = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{else.} \end{cases}$$



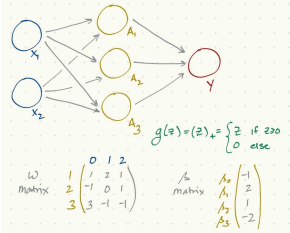
Matrix version: First layer



$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$

$$A = g(\mathbf{W} \cdot \mathbf{X}) \qquad \mathbf{X}^T = (1\ X_1\ X_2\ \cdots\ X_p)$$

Matrix version: Output



$$f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$

$$Y = \beta \cdot \mathbf{A} \qquad \mathbf{A}^T = (1\ A_1\ A_2\ \cdots\ A_K)$$

Now what?

Choose parameters by minimizing RSS, $\sum_{i=1}^{n}(y_i - f(x_i))^2$

Chosen in advance:

Tuned by the model:

Coding

Next time

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