Ch 10.2: Multi-Layer Neural Nets

Lecture 30 - CMSE 381

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Announcements

Last time:

- Single Layer Neural Nets

This lecture:

- Multi-layer Neural Nets
- Application to MNIST

Announcements:

- HW 9 Due Sunday 11/23
- Exam 3 11/24
- Project Due 12/5

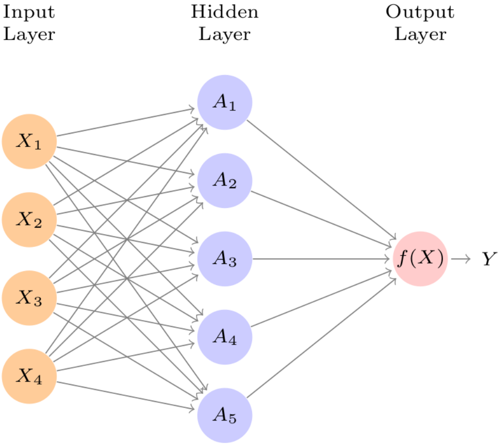
The end is near!

27	F	11/7	SVC	9.2	HW #7 Due Sun 11/9
28	M	11/10	SVM	9.3, 9.4	
29	W	11/12	Single Layer NN	10.1	
30	F	11/13	Multi Layer NN	10.2	HW #8 Due Sun 11/16
31	M	11/17	CNN	10.3	
32	W	11/19	Unsupervised learning / clustering	12.1, 12.4	
33	F	11/21	Virtual: Project Office Hours		HW #9 Due Sun 11/23
	M	11/24	***Review***		
	W	11/26	**Midterm #3**		
	F	11/28	Thanksgiving		
	M	12/1	Virtual: Project Office Hours		
	W	12/3	Virtual: Project Office Hours		
	F	12/5			**Project Due**

Section 1

Neural Nets

Feed Forward Neural Network: The cartoon



$$A_k = h_k(X) = g(w_{k0} + \textstyle\sum_{j=1}^{p} w_{kj} X_j),$$

$$f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$

Test your understanding: PollEv

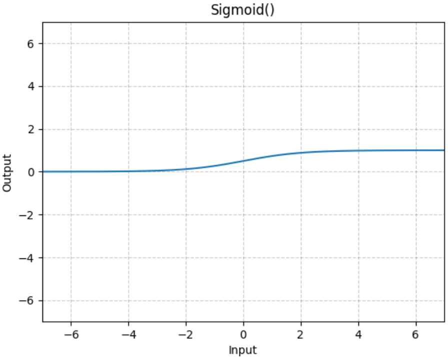
Choices for activation function

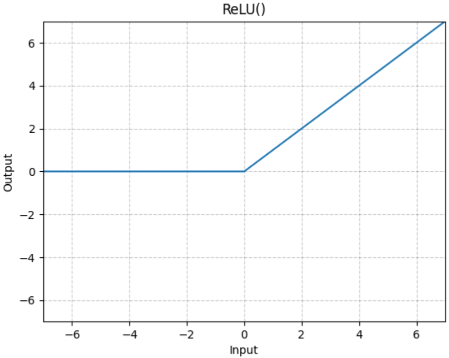
Sigmoid:

$$g(z) = \frac{e^z}{1 + e^z} = \frac{1}{1 + e^{-z}}$$

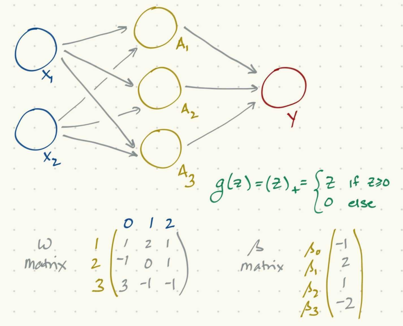
ReLU: Rectified linear unit

$$g(z) = (z)_+ = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{else.} \end{cases}$$





Matrix version



$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$

$$A = g(\mathbf{W} \cdot \mathbf{X}) \qquad \mathbf{X}^T = (1 \ X_1 \ X_2 \ \cdots \ X_p)$$

⋈⋈⋈⋈⋈⋈

$$f(X) = \beta_0 + \sum_{k=1}^{K} \beta_k A_k$$

$$Y = \beta \cdot \mathbf{A} \qquad \mathbf{A}^T = (1 \ A_1 \ A_2 \ \cdots \ A_K)$$

Training the model

Choose parameters by minimizing RSS, $\sum_{i=1}^{n}(y_i - f(x_i))^2$ (or other loss function)

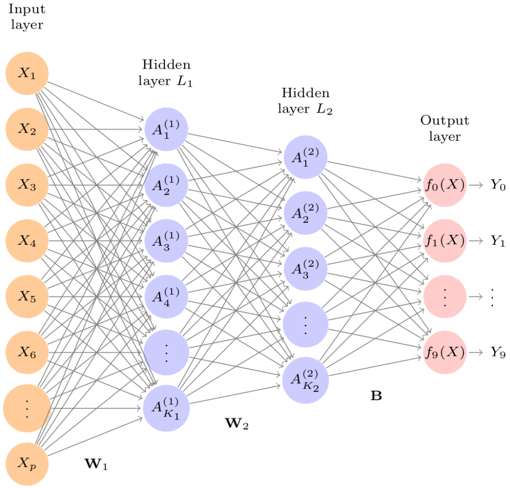
Chosen in advance:

Tuned by the model:

Section 2

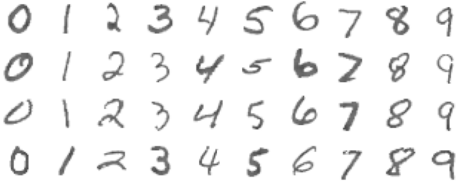
Multilayer Neural Networks

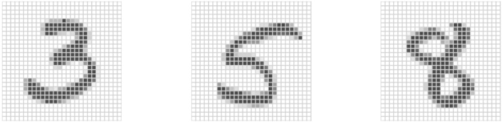
Multiple layers



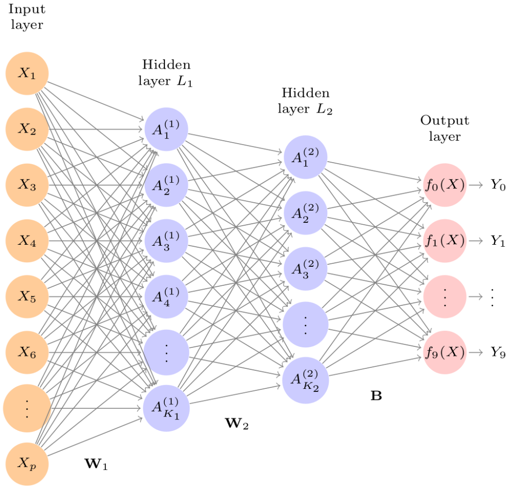
More typical for image classification

Example: MNIST





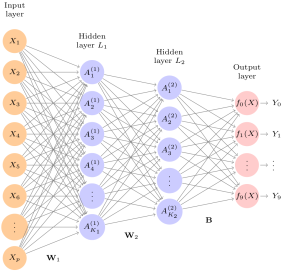
Hidden layers



$$
\begin{aligned}
A_k^{(1)} &= h_k^{(1)}(X) \\
&= g(w_{k0}^{(1)} + \textstyle\sum_{j=1}^{p} w_{kj}^{(1)} X_j)
\end{aligned}
$$

$$
\begin{aligned}
A_\ell^{(2)} &= h_\ell^{(2)}(X) \\
&= g(w_{\ell 0}^{(2)} + \textstyle\sum_{k=1}^{K_1} w_{\ell k}^{(2)} A_k^{(1)})
\end{aligned}
$$

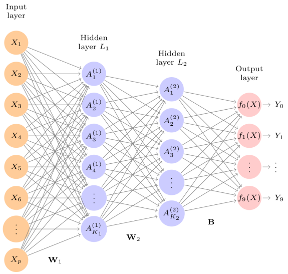
More on that architecture



$$\begin{aligned} A_k^{(1)} &= h_k^{(1)}(X) \\ &= g(w_{k0}^{(1)} + \textstyle\sum_{j=1}^{p} w_{kj}^{(1)} X_j) \end{aligned}$$

$$\begin{aligned} A_\ell^{(2)} &= h_\ell^{(2)}(X) \\ &= g(w_{\ell 0}^{(2)} + \textstyle\sum_{k=1}^{K_1} w_{\ell k}^{(2)} A_k^{(1)}) \end{aligned}$$

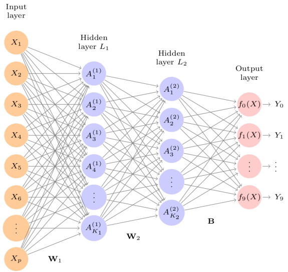
Matrix version: First layer



$$
\begin{aligned}
A_k^{(1)} &= h_k^{(1)}(X) \\
&= g(w_{k0}^{(1)} + \textstyle\sum_{j=1}^{p} w_{kj}^{(1)} X_j)
\end{aligned}
$$

$$A^{(1)} = g(\mathbf{W}^{(1)} \cdot \mathbf{X}) \qquad \mathbf{X}^T = (1 \; X_1 \; X_2 \; \cdots \; X_p)$$

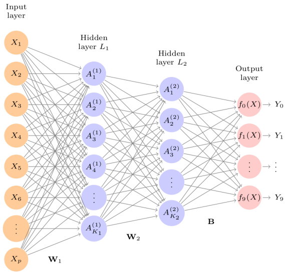
Matrix version: Second layer



$$
\begin{aligned}
A_\ell^{(2)} &= h_\ell^{(2)}(X) \\
&= g\left(w_{\ell 0}^{(2)} + \textstyle\sum_{k=1}^{K_1} w_{\ell k}^{(2)} A_k^{(1)}\right)
\end{aligned}
$$

$$A^{(2)} = g(\mathbf{W}^{(2)} \cdot \mathbf{A}^{(1)}) \qquad (\mathbf{A}^{(1)})^T = (1 \; A_1^{(1)} \; A_2^{(1)} \; \cdots \; A_{K_1}^{(1)})$$

Matrix version: Last layer, first step

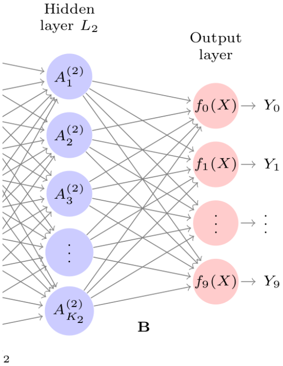


$$
\begin{aligned}
Z_m &= \beta_{m0} + \textstyle\sum_{\ell=1}^{K_2} \beta_{m\ell} h_\ell^{(2)}(X) \\
&= \beta_{m0} + \textstyle\sum_{\ell=1}^{K_2} \beta_{m\ell} A_\ell^{(2)},
\end{aligned}
$$

$$\mathbf{Z} = \beta \cdot \mathbf{A}^{(2)}$$

β is $M \times (K_2 + 1)$ matrix $\qquad (\mathbf{A}^{(2)})^T = (1 \; A_1^{(2)} \; A_2^{(2)} \; \cdots \; A_{K_2}^{(2)})$

The last column for classification: Softmax



$$f_m(X) = \Pr(Y = m|X) = \frac{e^{Z_m}}{\sum_{\ell=0}^{9} e^{Z_\ell}},$$

An example

$$f_m(X) = \Pr(Y = m|X) = \frac{e^{Z_m}}{\sum_{\ell=0}^{9} e^{Z_\ell}},$$

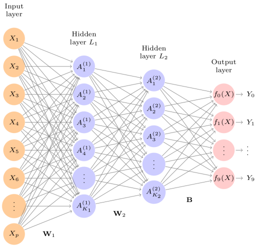
$$Z = \begin{pmatrix} 1 & 3 & -1 & 2 & 5 \end{pmatrix}$$

MNIST

Coding

TL;DR

Feed Forward Neural Net



$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$

- Combines input data using learned weights
- Linear combo of those to get output
- Sometimes softmax to get probability of classification

Next time

	F	10/17	***Review***		
	M	10/20	Fall Break		
	W	10/22	**Midterm #2**		
21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/28
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	
24	F	10/31	Decision Trees	8.1	HW #6 Due Sun 11/2
25	M	11/3	Random Forests	8.2.1, 8.2.2	
26	W	11/5	Maximal Margin Classifier	9.1	
27	F	11/7	SVC	9.2	HW #7 Due Sun 11/9
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Q of the day: How do you tell if an artificial neural network is for classification or regression? (if you only have the math)