Ch 8.1: Decision Trees

Lecture 24 - CMSE 381

Prof. Guanqun Cao

Michigan State University
::
Dept of Computational Mathematics, Science & Engineering

Wed, Oct 29, 2025

Announcements

Last time:

- Cubic Splines

This lecture:

- 8.1 Decision Trees

Announcements:

- HW #7 due Sun, 11/2
- Projects

	F	10/17	***Review***		
	M	10/20	Fall Break		
	W	10/22	**Midterm #2**		
21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/28
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	
24	F	10/31	Decision Trees	8.1	HW #6 Due Sun 11/2
25	M	11/3	Random Forests	8.2.1, 8.2.2	
26	W	11/5	Maximal Margin Classifier	9.1	
27	F	11/7	SVC	9.2	HW #7 Due Sun 11/9
28	M	11/10	SVM	9.3, 9.4	
29	W	11/12	Single Layer NN	10.1	
30	F	11/13	Multi Layer NN	10.2	HW #8 Due Sun 11/16
31	M	11/17	CNN	10.3	
32	W	11/19	Unsupervised learning / clustering	12.1, 12.4	
33	F	11/21	Virtual: Project Office Hours		HW #9 Due Sun 11/23
	M	11/24	***Review***		
	W	11/26	**Midterm #3**		
	F	11/28	Thanksgiving		
	M	12/1	Virtual: Project Office Hours		
	W	12/3	Virtual: Project Office Hours		
	F	12/5			**Project Due**

Section 1

Decision Trees

Big idea

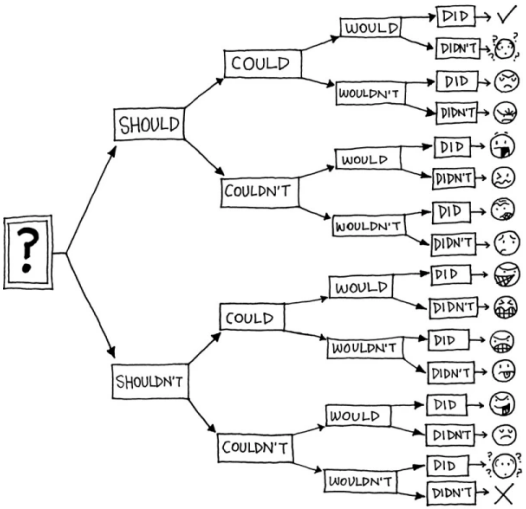


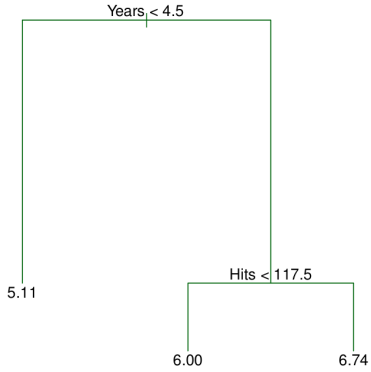
Image: https://marekbennett.com/2014/02/14/decision-tree/

Subset of Hitters data

	Hits	Years	Salary	LogSalary
1	81	14	475.0	6.163315
2	130	3	480.0	6.173786
3	141	11	500.0	6.214608
4	87	2	91.5	4.516339
5	169	11	750.0	6.620073
...	...	...	...	...
317	127	5	700.0	6.551080
318	136	12	875.0	6.774224
319	126	6	385.0	5.953243
320	144	8	960.0	6.866933
321	170	11	1000.0	6.907755

First decision tree example

	Hits	Years	LogSalary
1	81	14	6.163315
2	130	3	6.173786
3	141	11	6.214608
4	87	2	4.516339
5	169	11	6.620073
...	...	...	...
317	127	5	6.551080
318	136	12	6.774224
319	126	6	5.953243
320	144	8	6.866933
321	170	11	6.907755

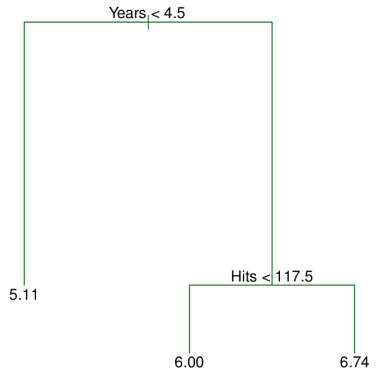


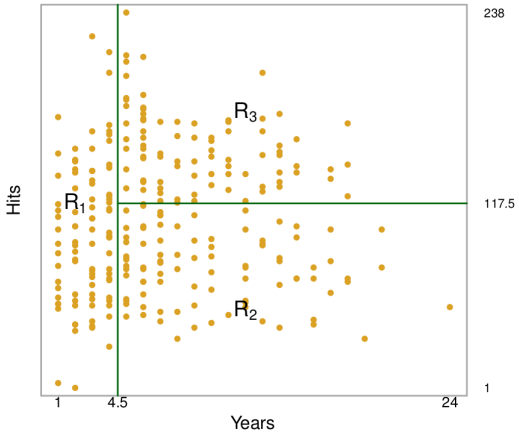
Interpretation of example

Coding a regression decision tree

Regions defined by the tree

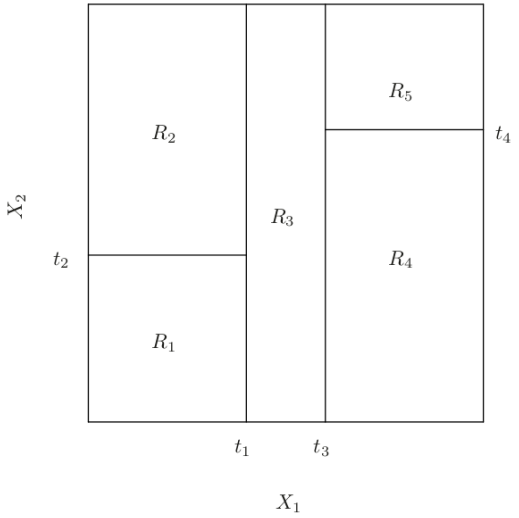
Viewing Regions Defined by Tree





How do we actually get the tree? Two steps

1. We divide the predictor space – that is, the set of possible values for $X_1, X_2, \cdots, X_p$ — into J distinct and non-overlapping regions, $R_1, R_2, \cdots, R_J$.
2. For every observation that falls into the region R_j, we make the same prediction = the mean of the response values for the training observations in R_j.



Step 1: How do we decide on R_js?

Goal:
Find boxes $R_1, \cdots, R_J$ that minimize

$$\sum_{j=1}^{J} \sum_{i \in R_j} (y_i - \hat{y}_{R_j})^2$$

$\hat{y}_{R_j}$ = mean response for training observations in jth box

Recursive Binary Splitting

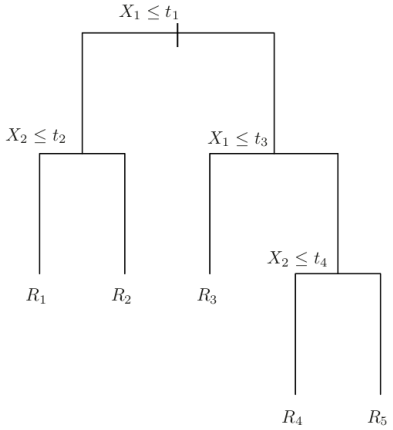
One split:

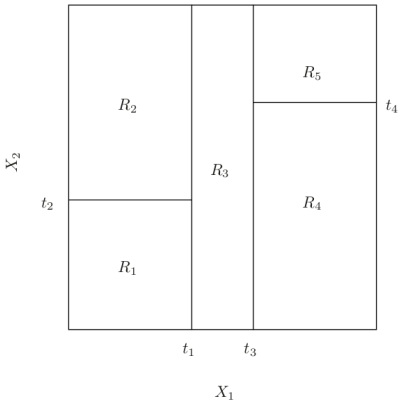
- Pick X_j and cutpoint s
- so that splitting into $\{X \mid X_j < s\}$ and $\{X \mid X_j \geq s\}$ results in largest possible reduction in RSS

$$R_1(j, s) = \{X \mid X_j < s\}$$
$$R_2(j, s) = \{X \mid X_j \geq s\}$$

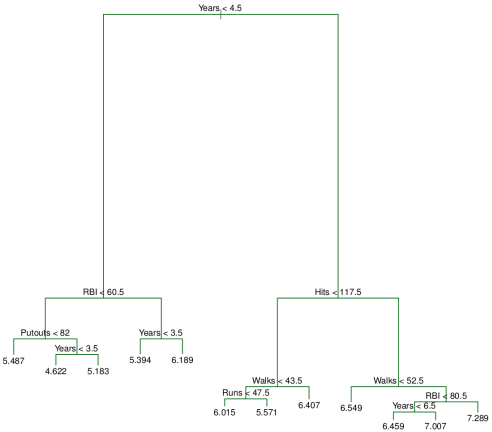
$$\sum_{i|x_i \in R_1(j,s)} (y_i - \hat{y}_{R_1})^2 + \sum_{i|x_i \in R_2(j,s)} (y_i - \hat{y}_{R_2})^2$$

Rinse and repeat

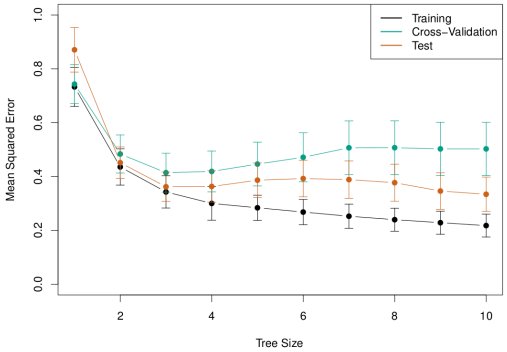


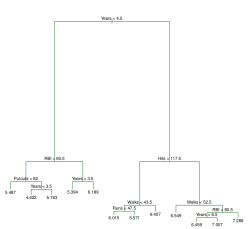


Pruning


Years < 4.5
RBI < 60.5
Hits < 117.5
Putouts < 82
Years < 3.5
Years < 3.5
Walks < 43.5
Walks < 52.5
5.487
Years < 3.5
5.394
6.189
Runs < 47.5
RBI < 80.5
6.015
5.571
6.407
6.549
Years < 6.5
4.622
5.183
6.459
7.007
7.289

Result of pruning

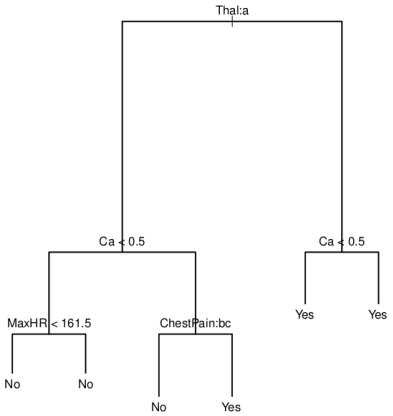




Section 2

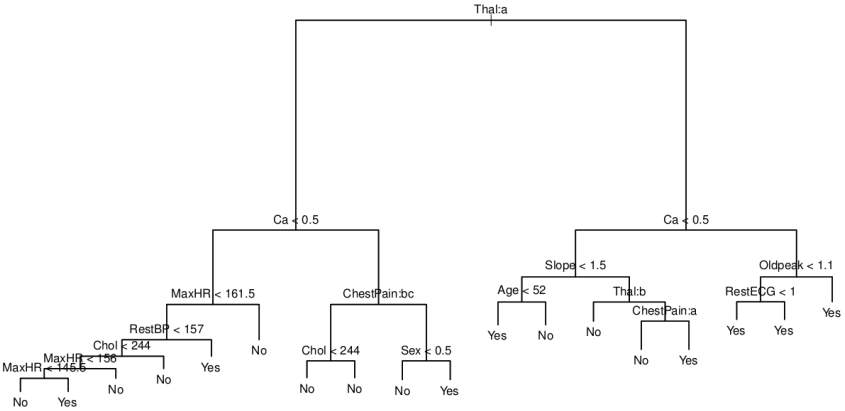
Classification Decision Tree

Basic idea

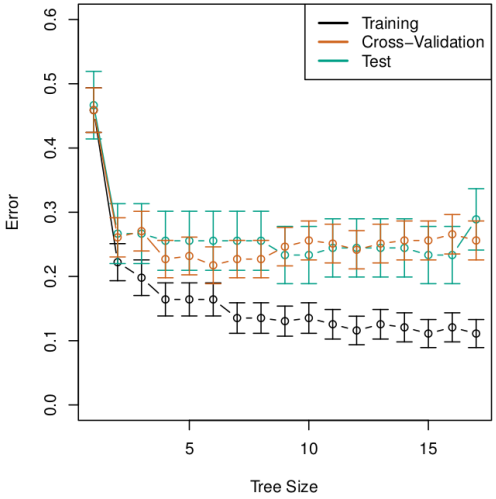


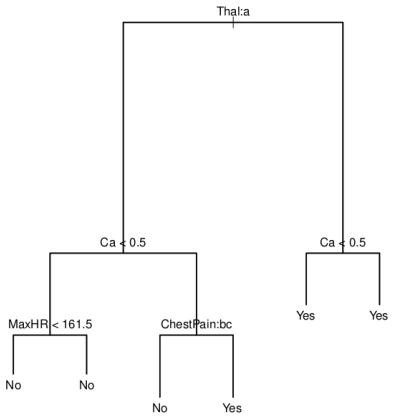
- $\hat{p}_{mk}$ = proportion of training observations in R_m from the kth class
- $E = 1 - \max_k(\hat{p}_{mk})$

Example


Thal:a
Ca < 0.5
Ca < 0.5
MaxHR < 161.5
ChestPain:bc
Slope < 1.5
Oldpeak < 1.1
Age < 52
Thal:b
RestECG < 1
RestBP < 157
Chol < 244
MaxHR < 156
MaxHR < 145.5
Chol < 244
Sex < 0.5
ChestPain:a
No
Yes
No
Yes
No
Yes
No
No
No
Yes
No
Yes
No
Yes
No
Yes
Yes
Yes
No
Yes
No
Yes
No
Yes

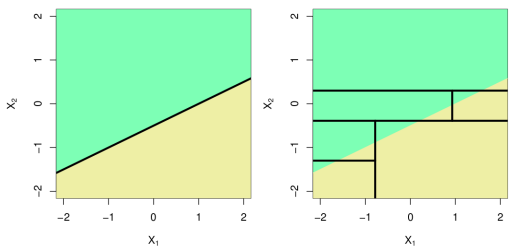
Pruning the example

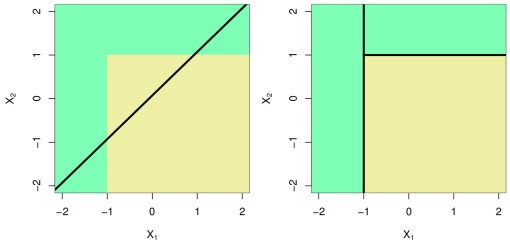




More coding!

Linear models vs trees





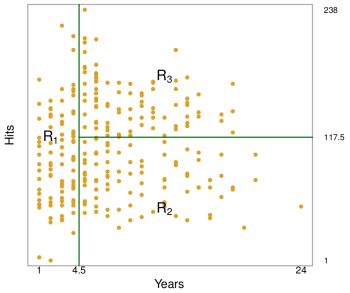
Pros/Cons

Pros:

Cons:

TL;DR

- Split into regions by greedily decreasing RSS
- Prune tree by using cost complexity
- Not robust - Next time, figure out how to aggregate trees



Next time

	F	10/17	***Review***		
	M	10/20	Fall Break		
	W	10/22	**Midterm #2**		
21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/28
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	
24	F	10/31	Decision Trees	8.1	HW #6 Due Sun 11/2
25	M	11/3	Random Forests	8.2.1, 8.2.2	
26	W	11/5	Maximal Margin Classifier	9.1	
27	F	11/7	SVC	9.2	HW #7 Due Sun 11/9
28	M	11/10	SVM	9.3, 9.4	
29	W	11/12	Single Layer NN	10.1	
30	F	11/13	Multi Layer NN	10.2	HW #8 Due Sun 11/16
31	M	11/17	CNN	10.3	
32	W	11/19	Unsupervised learning / clustering	12.1, 12.4	
33	F	11/21	Virtual: Project Office Hours		HW #9 Due Sun 11/23
	M	11/24	***Review***		
	W	11/26	**Midterm #3**		
	F	11/28	Thanksgiving		
	M	12/1	Virtual: Project Office Hours		
	W	12/3	Virtual: Project Office Hours		