Ch 8.2.1, 8.2.2: Bagging and Random Forests

Lecture 25 - CMSE 381

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Announcements

Last time:

- 8.1 Decision Trees - regression

This lecture:

- 8.1 Decision Trees - classification
- 8.2.1 Bagging
- 8.2.2 Random forest

Announcements:

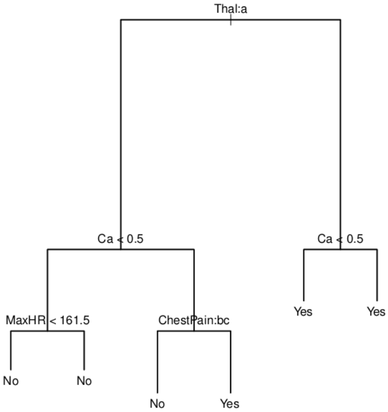
- Homework 7 Due Sunday

	F	10/17	***Review***		
	M	10/20	Fall Break		
	W	10/22	**Midterm #2**		
21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/28
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	
24	F	10/31	Decision Trees	8.1	HW #6 Due Sun 11/2
25	M	11/3	Random Forests	8.2.1, 8.2.2	
26	W	11/5	Maximal Margin Classifier	9.1	
27	F	11/7	SVC	9.2	HW #7 Due Sun 11/9
28	M	11/10	SVM	9.3, 9.4	
29	W	11/12	Single Layer NN	10.1	
30	F	11/13	Multi Layer NN	10.2	HW #8 Due Sun 11/16
31	M	11/17	CNN	10.3	
32	W	11/19	Unsupervised learning / clustering	12.1, 12.4	
33	F	11/21	Virtual: Project Office Hours		HW #9 Due Sun 11/23
	M	11/24	***Review***		
	W	11/26	**Midterm #3**		
	F	11/28	Thanksgiving		
	M	12/1	Virtual: Project Office Hours		
	W	12/3	Virtual: Project Office Hours		
	F	12/5			**Project Due**

Section 1

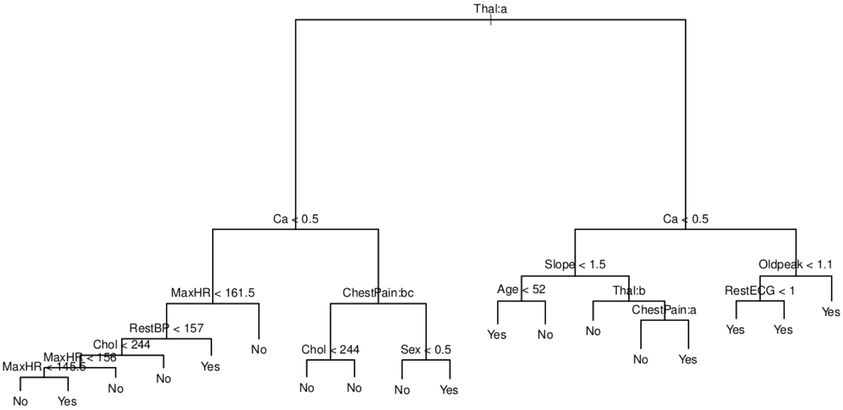
Classification Decision Tree

Basic idea

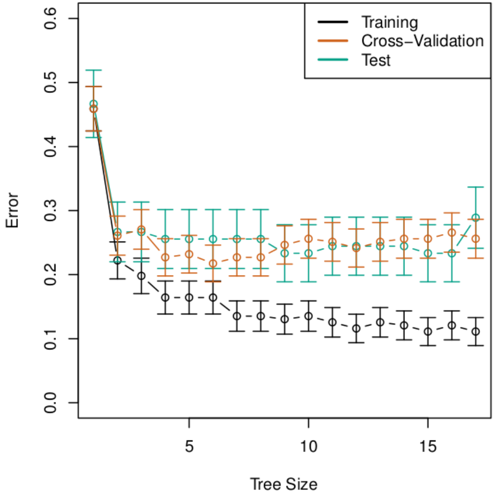


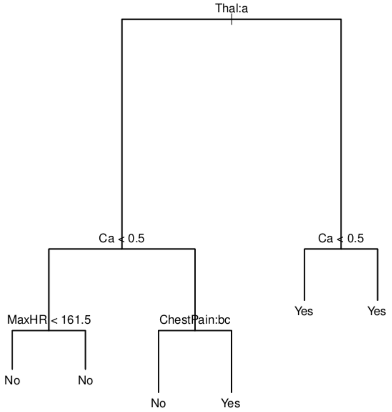
- $\hat{p}_{mk}$ = proportion of training observations in R_m from the kth class
- $E = 1 - \max_k(\hat{p}_{mk})$

Example


Thal:a
Ca < 0.5
Ca < 0.5
MaxHR < 161.5
ChestPain:bc
Slope < 1.5
Oldpeak < 1.1
Age < 52
Thal:b
RestECG < 1
ChestPain:a
RestBP < 157
Chol < 244
MaxHR < 156
MaxHR < 145.5
Chol < 244
Sex < 0.5
No
Yes
No
No
Yes
No
No
No
No
Yes
Yes
No
No
No
Yes
Yes
Yes
Yes

Pruning the example

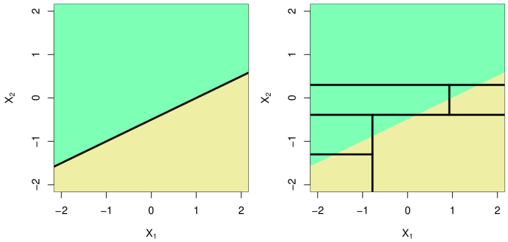


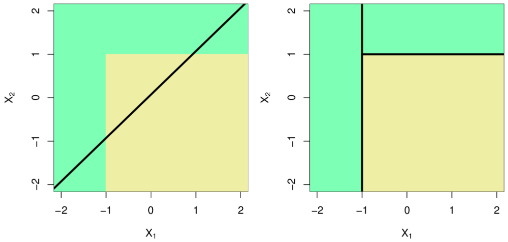


Coding!

Second part of day 24's jupyter notebook.

Linear models vs trees





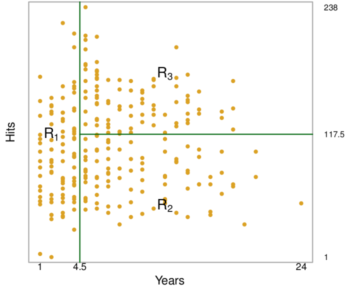
Pros/Cons

Pros:

Cons:

TL;DR

- Split into regions by greedily decreasing RSS (or error rate)
- Prune tree by using cost complexity
- Not robust - Next, figure out how to aggregate trees



Section 2

8.2.1 Bagging

The bootstrap

Want to do (but can't):
Build separate models from independent training sets, and average resulting predictions:

- $\hat{f}^1(x), \cdots, \hat{f}^B(x)$ for B separate training sets
- Return the average

$$\hat{f}_{avg}(x) = \frac{1}{B}\sum_{b=1}^{B} \hat{f}^b(x)$$

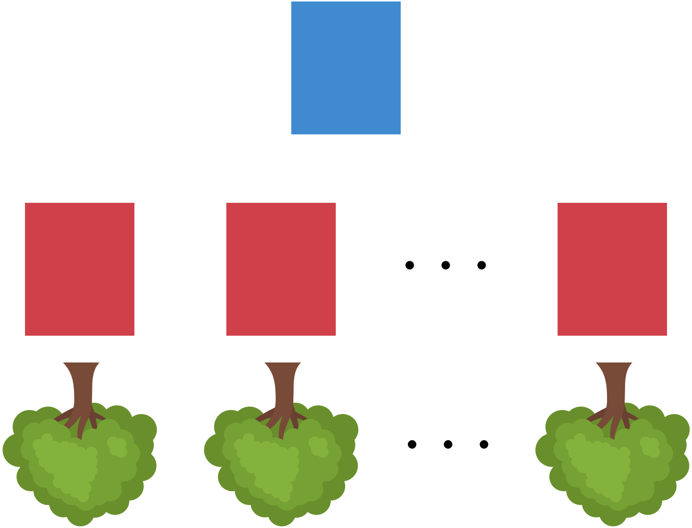
Boostrap modification:

- Work with fixed data set
- Take B samples from this data set (with replacement)
- Train method on bth sample to get $\hat{f}^{*b}(x)$
- Return average of predictions (regression)

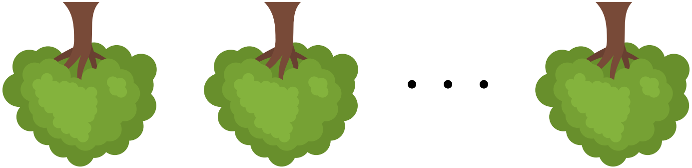
$$\hat{f}_{bag}(x) = \frac{1}{B}\sum_{b=1}^{B} \hat{f}^{*b}(x)$$

or majority vote (classification)

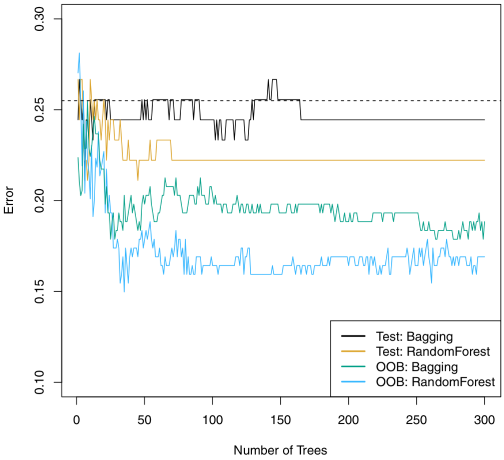
Tree version



Prediction on new data point

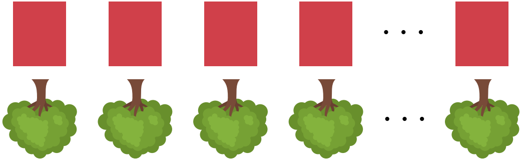


Example: Heart classification data

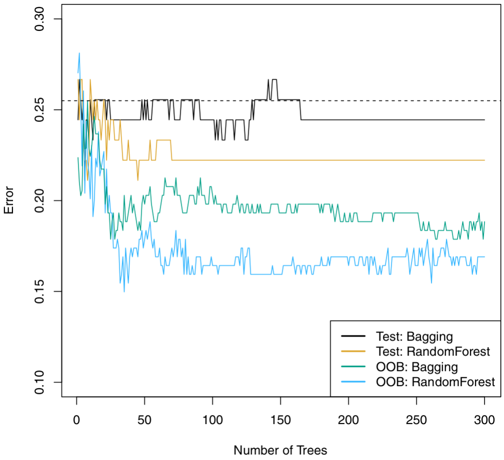


Out of Bag Error Estimation

- On average, bootstrap sample uses about 2/3 of the data
- Remaining observations not used are called *out-of-bag* (OOB) observations
- For each observation, run through all the trees where it wasn't used for building
- Return the average (or majority vote) of those as test prediction



Error using OOB



Section 3

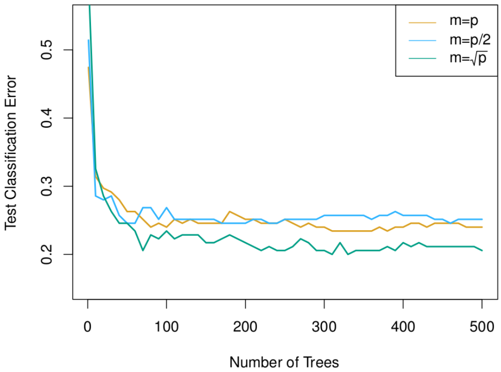
Random Forests

The idea

- Goal is to decorrelate the bagged trees:
    - If there is a strong predictor, the first split of most trees will be the same
    - Most or all trees will be highly correlated
    - Averaging highly correlated quantities doesn't decrease variance as much as uncorrelated

- The random forest fix:
    - Each time a split is considered, only use a random subset of m the predictors
    - Fresh sample taken every time
    - Typically $m \approx \sqrt{p}$
    - On average, $(p - m)/p$ of splits won't consider strong predictor
    - $m = p$ gives back bagging

Example on gene expression



Coding time!

TL:DR

- Bagging: trees grown independently on random samples. Trees tend to be similar to each other, can result in getting caught in local optima
- Random forest: trees independently on samples, but split is done using random subset of features

Next time

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Q of the day:
You have two very different datasets to create two very different models.
You have to use random forest on one and bagging on the other.
Which one would benefit more from random forest? what criteria would you use for the making the decision?