Ch 10.3: Convolutional Neural Nets

Lecture 31 - CMSE 381

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Monday Nov 17, 2025

Announcements

Last time:

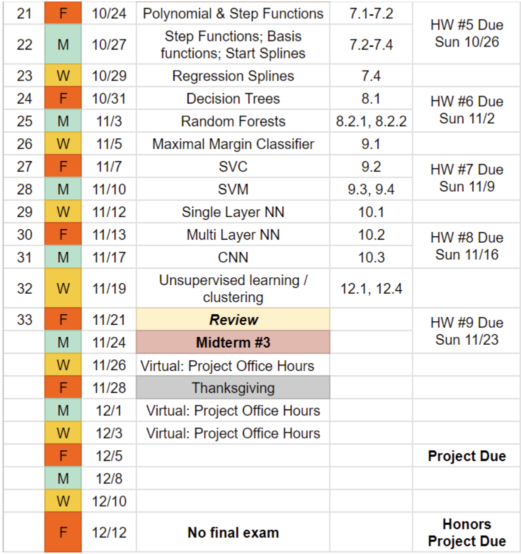
- Multilayer NN
- pyTorch

This lecture:

- CNNs

Final countdown:

- Exam 3 is 11/24
- Project is due 12/5



21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/26
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33	F	11/21	***Review***		HW #9 Due Sun 11/23
	M	11/24	**Midterm #3**		
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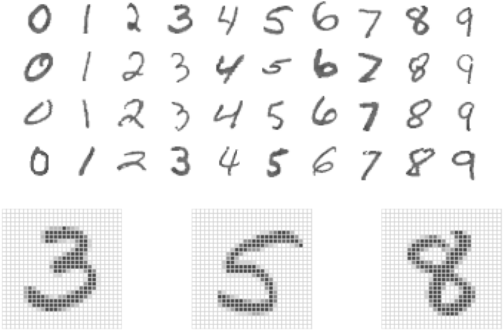
	W	12/3	Virtual: Project Office Hours		
	F	12/5			**Project Due**
	M	12/8			
	W	12/10			
	F	12/12	**No final exam**		**Honors Project Due**

Section 1

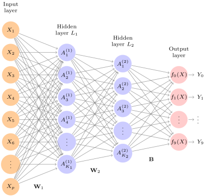
Last time: Neural Nets

MNIST



- Goal: Build a model to classify images into their correct digit class
- Each image has $p = 28 \cdot 28 = 784$ pixels
- Each pixel is grayscale value in [0,255]
- Data converted into column order
- Output represented by one-hot vector $Y = (Y_0, Y_1, \cdots, Y_9)$
- 60K training images, 10K test images

Neural network architecture for MNIST



$$
\begin{aligned}
A_k^{(1)} &= h_k^{(1)}(X) \\
&= g(w_{k0}^{(1)} + \textstyle\sum_{j=1}^{p} w_{kj}^{(1)} X_j)
\end{aligned}
$$

$$
\begin{aligned}
A_\ell^{(2)} &= h_\ell^{(2)}(X) \\
&= g(w_{\ell 0}^{(2)} + \textstyle\sum_{k=1}^{K_1} w_{\ell k}^{(2)} A_k^{(1)})
\end{aligned}
$$

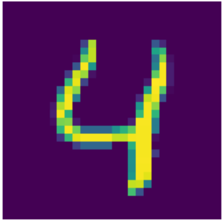
$$
\begin{aligned}
Z_m &= \beta_{m0} + \textstyle\sum_{\ell=1}^{K_2} \beta_{m\ell} h_\ell^{(2)}(X) \\
&= \beta_{m0} + \textstyle\sum_{\ell=1}^{K_2} \beta_{m\ell} A_\ell^{(2)},
\end{aligned}
$$

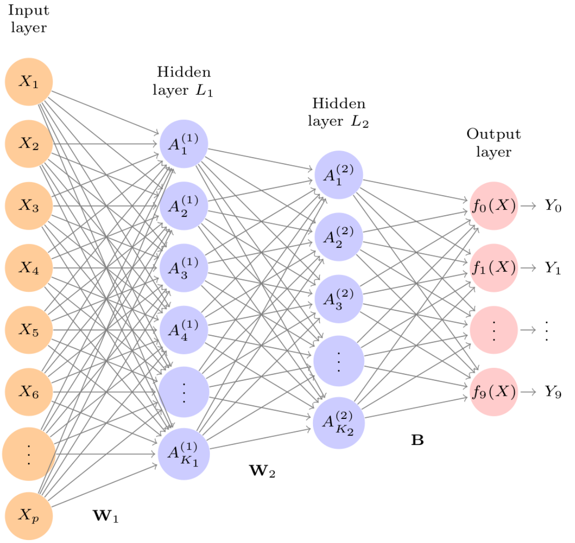
$$
f_m(X) = \Pr(Y = m|X) = \frac{e^{Z_m}}{\sum_{\ell=0}^{9} e^{Z_\ell}},
$$

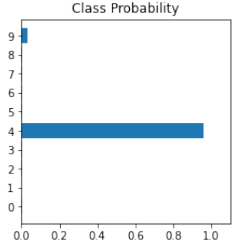
- Two hidden layers.
- Softmax for classification output
- We used L_1 has 128 units; L_2 has 64
- 10 output variables due to class labeling
- Result is we are training approx 110K weights

Test your understanding: PollEv

MNIST learning







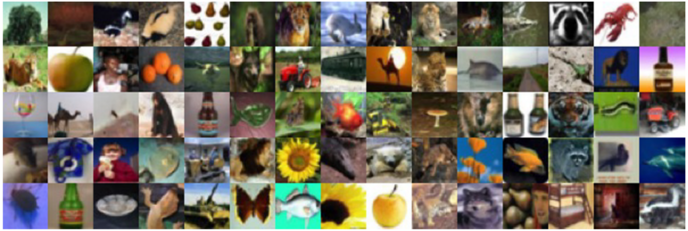
Section 2

Convolutional Neural Network

Last time: Flattening the image

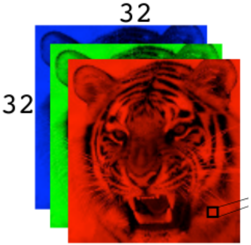
$$\begin{pmatrix} 1 & 1 & 0 \\ 4 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \\ 4 \\ 2 \\ 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}$$

Example data set: CIFAR100 Data

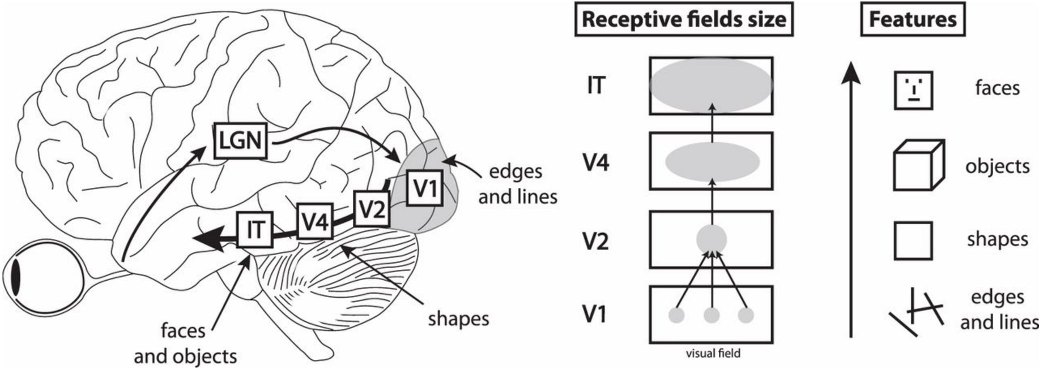


- 60,000 images: 50K training, 10K test
- Labels with 20 super classes (e.g. aquatic mammals)
- 5 classes per super class (beaver, dolphin, otter, seal, whale)
- Images are 32x32

Image channel data

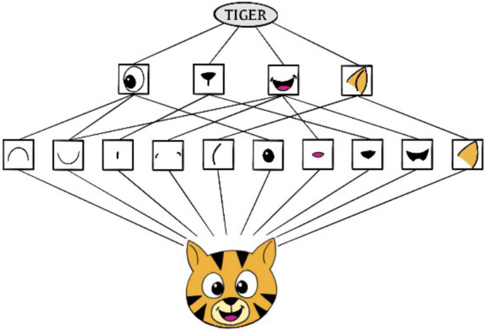


How does your brain do it? The visual hierarchy



Mauro Manassi, Bilge Sayim, Michael H. Herzog; When crowding of crowding leads to uncrowding. Journal of Vision 2013;13(13):10. https://doi.org/10.1167/13.13.10.

CNNs



Convolution layer

Convolution Filter

Original Image:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \\ j & k & l \end{bmatrix}$$

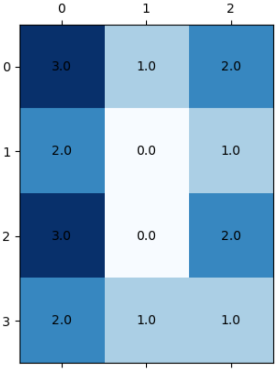
Convolution filter:

$$\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$$

Convolved Image

$$\begin{bmatrix} a\alpha + b\beta + d\gamma + e\delta & b\alpha + c\beta + e\gamma + f\delta \\ d\alpha + e\beta + g\gamma + h\delta & e\alpha + f\beta + h\gamma + i\delta \\ g\alpha + h\beta + j\gamma + k\delta & h\alpha + i\beta + k\gamma + l\delta \end{bmatrix}$$

Convolution Filter Example



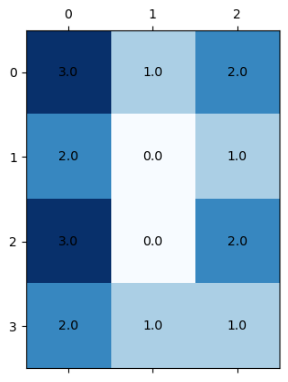
	0	1	2
0	3.0	1.0	2.0
1	2.0	0.0	1.0
2	3.0	0.0	2.0
3	2.0	1.0	1.0

Filter:

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Same example, different filter

What is the convolved image?



	0	1	2
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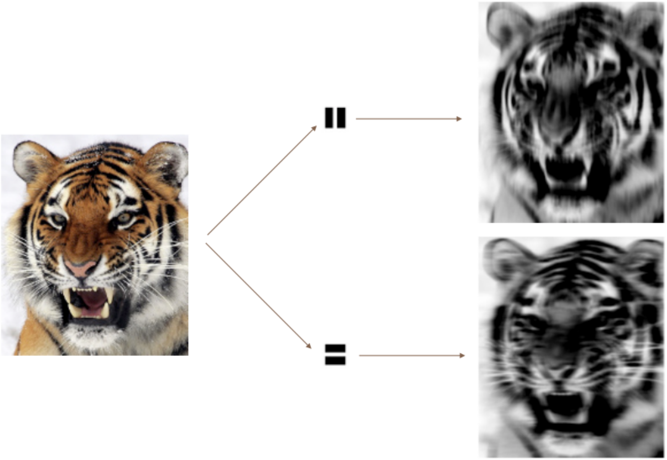
1	2.0	0.0	1.0
2	3.0	0.0	2.0
3	2.0	1.0	1.0

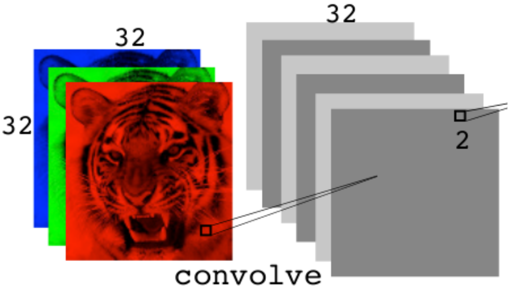
Filter:

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Convolution filter: Bigger example



Convolution layer

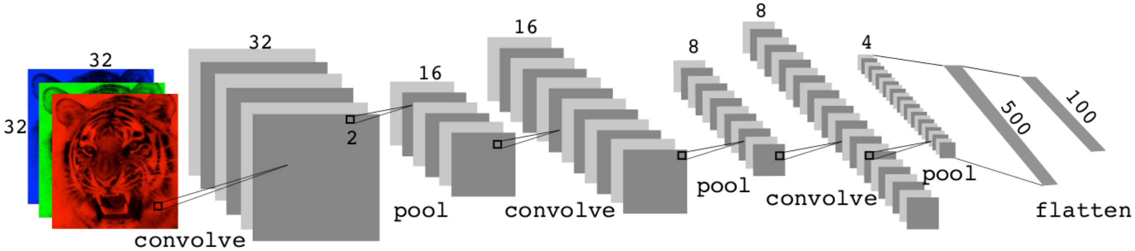


More notes on convolution

Pooling layers

$$\text{Max pool } \begin{bmatrix} 1 & 2 & 5 & 3 \\ 3 & 0 & 1 & 2 \\ 2 & 1 & 3 & 4 \\ 1 & 1 & 2 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 & 5 \\ 2 & 4 \end{bmatrix}$$

Putting it together to make a CNN

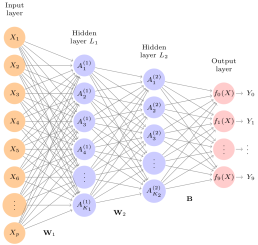


https://poloclub.github.io/cnn-explainer/

Coding

TL;DR

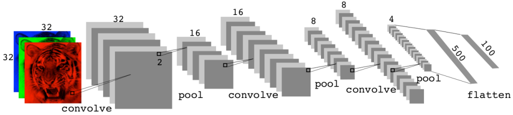
Feed Forward Neural Net



$$A_k = h_k(X) = g(w_{k0} + \sum_{j=1}^{p} w_{kj} X_j),$$

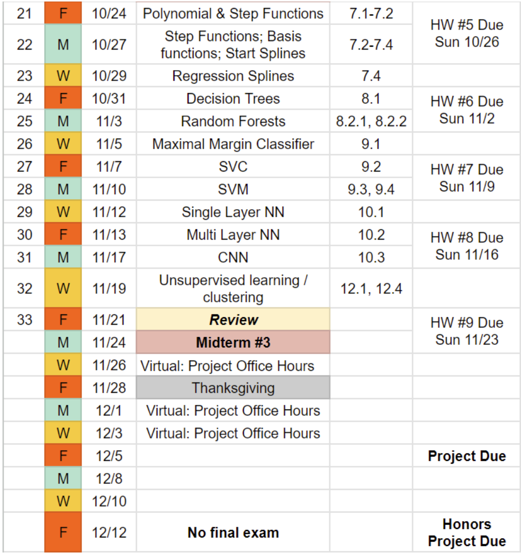
- Combines input data using learned weights
- Linear combo of those to get output
- Sometimes softmax to get probability of classification

CNN



- Specialized NN
- Gets next layer via
  - Convolution layer
  - Pooling Layer
  - Fully connected layer

Next time



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