Ch 7.1-7.2: Polynomial regression and Step Functions

Lecture 21 - CMSE 381

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Announcements

Last time:

- Exam 2

This lecture:

- 7.1 Polynomial regression
- 7.2 Step functions

Announcements:

- HW #6 due Sun 11/2
- Project stuff

21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/26
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	
24	F	10/31	Decision Trees	8.1	HW #6 Due Sun 11/2
25	M	11/3	Random Forests	8.2.1, 8.2.2	
26	W	11/5	Maximal Margin Classifier	9.1	
27	F	11/7	SVC	9.2	HW #7 Due Sun 11/9
28	M	11/10	SVM	9.3, 9.4	
29	W	11/12	Single Layer NN	10.1	
30	F	11/13	Multi Layer NN	10.2	HW #8 Due Sun 11/16
31	M	11/17	CNN	10.3	
32	W	11/19	Unsupervised learning / clustering	12.1, 12.4	
33	F	11/21	***Review***		HW #9 Due Sun 11/23
	M	11/24	**Midterm #3**		
	W	11/26	Virtual: Project Office Hours		
	F	11/28	Thanksgiving		
	M	12/1	Virtual: Project Office Hours		
	W	12/3	Virtual: Project Office Hours		
	F	12/5			**Project Due**
	M	12/8			
	W	12/10			
	F	12/12	**No final exam**		**Honors Project Due**

What should you learn from today's lecture?

- Why do people want to use polynomial regression rather than simple linear regression
- How to fit polynomial regression models?
- Why do people want to use step functions rather than simple linear regression?
- How to fit step function models?
- What is common between polynomial regression and step functions?
- What is different between polynomial regression and step functions?

Section 1

Polynomial Regression

Polynomial regression

Replace linear model

$$y_i = \beta_0 + \beta_1 x_1 + \varepsilon_i$$

with

$$y_i = \beta_0 + \beta_1 x_1 + \beta_2 x_i^2 + \cdots + \beta_d x_i^d + \varepsilon_i$$

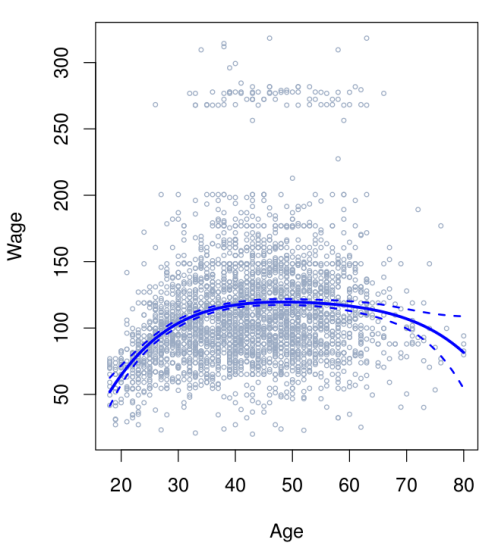
Example: Wage Data

$$\texttt{wage} = \beta_0 + \beta_1 \texttt{age} + \beta_2 \texttt{age}^2 + \cdots + \beta_p \texttt{age}^p + \varepsilon.$$

My code learned:

$$-184.1542 + 21.24552 \cdot age + -0.56386 \cdot age^2 + 0.00681 \cdot age^3 + (-3 \cdot 10^{-5}) \cdot age^4$$

Example with wage data



Section 2

Step function

Step functions

$$\mathrm{I}(X < c) \qquad \mathrm{I}(c_1 \leq X < c_2) \qquad \mathrm{I}(c \leq X)$$

More on step function setup

$$
\begin{aligned}
C_0(X) &= I(X < c_1), \\
C_1(X) &= I(c_1 \le X < c_2), \\
C_2(X) &= I(c_2 \le X < c_3), \\
&\vdots \\
C_{K-1}(X) &= I(c_{K-1} \le X < c_K), \\
C_K(X) &= I(c_K \le X),
\end{aligned}
$$

Test your understanding: PollEv

Example

Given knots $c_1 = 3$, $c_2 = 5$, $c_3 = 7$, determine the entries in the columns for $C_i(X)$ in the below matrix.

x	$C_0(X)$	$C_1(X)$	$C_2(X)$	$C_3(X)$
1				
2				
3				
4				
5				

x	$C_0(X)$	$C_1(X)$	$C_2(X)$	$C_3(X)$
6				
7				
8				
9				
10				

Draw the function

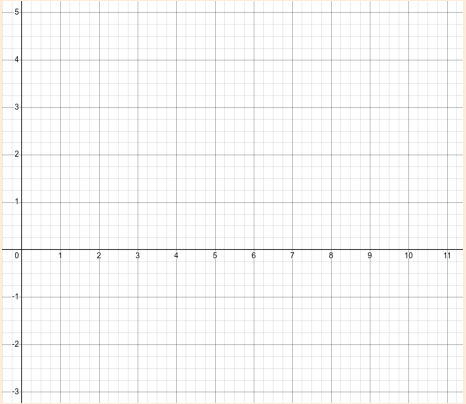
My code doing regression on the step function input returned the function.

$$f(X) = -1 + 3C_1(X) + 4C_2(X) - 2C_3(X).$$

Fill in the table of values, then draw this function below.

X	$F(X)$
1	
2	
3	
4	
5	

X	$F(X)$
6	
7	
8	
9	
10	



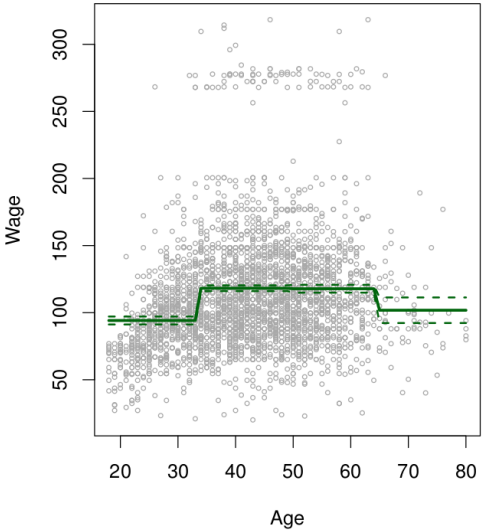
Step function: Learned model

$$y_i = \beta_0 + \beta_1 C_1(x_i) + \beta_2 C_2(x_i) + \cdots + \beta_K C_K(x_i) + \varepsilon_i$$

Coding bit

Back to the `wage` data set

Step function example



Next time

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