Ch 3.1: Linear Regression

Lecture 4 - CMSE 381

Prof. Guanqun Cao

Michigan State University
::
Dept of Computational Mathematics, Science & Engineering

Wed Sep 3, 2025

Announcements

Last time:

- 2.2 Assessing Model Accuracy

Covered in this lecture

- Least squares coefficient estimates for linear regression
- Residual sum of squares (RSS)

Section 1

Simple Linear Regression

Setup

- Predict Y on a single predictor variable X

$$Y \approx \beta_0 + \beta_1 X$$

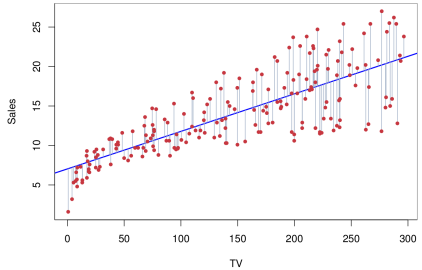
- "$\approx$" .... "is approximately modeled as"

Example

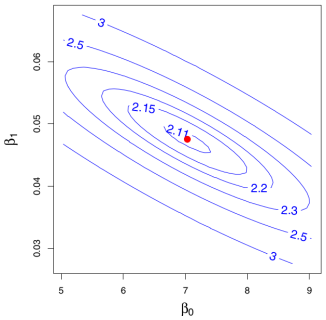
	TV	Radio	Newspaper	Sales
1	230.1	37.8	69.2	22.1
2	44.5	39.3	45.1	10.4
3	17.2	45.9	69.3	9.3
4	151.5	41.3	58.5	18.5
5	180.8	10.8	58.4	12.9
6	8.7	48.9	75	7.2
7	57.5	32.8	23.5	11.8
8	120.2	19.6	11.6	13.2
9	8.6	2.1	1	4.8
10	199.8	2.6	21.2	10.6
11	66.1	5.8	24.2	8.6

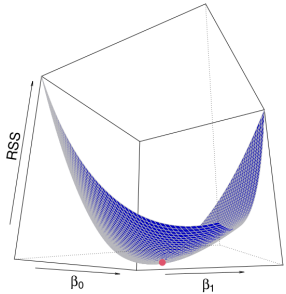
Least squares criterion: Setup

How do we estimate the coefficients?



Least squares criterion: RSS





Residual sum of squares RSS is

$$\begin{aligned} RSS &= e_1^2 + \cdots + e_n^2 \\ &= \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2 \end{aligned}$$

$$\text{sales} \approx \beta_0 + \beta_1 \text{TV}$$

Least squares criterion

Find β_0 and β_1 that minimize the RSS.

Least squares coefficient estimates

$$\min_{\beta_0,\beta_1} \sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2$$

$$\frac{\partial RSS}{\partial \hat{\beta}_0} = -2\sum_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

$$\frac{\partial RSS}{\partial \hat{\beta}_1} = -2\sum_i x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) = 0$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sum_{i=1}^n (x_i - \overline{x})^2}$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x}$$

Coding group work

Next time

Next time:

- More on simple linear regression!
- Evaluation of model etc.

CMSE381_F2025_Schedule : Schedule

Lec #	Date		Topic	Reading	HW
1	M	8/25	Intro / Python Review	1	
2	W	8/27	What is statistical learning	2.1	
3	F	8.29	Assessing Model Accuracy	2.2.1, 2.2.2	
	M	9/1	Labor Day - No Class		
4	W	9/3	Linear Regression	3.1	
5	F	9/5	More Linear Regression	3.1	HW #1 Due Sun 9/7
6	M	9/8	Multi-linear Regression	3.2	
7	W	9/10	Probably More Linear Regression	3.3	
8	F	9/12	Last of the Linear Regression		HW #2 Due

Announcements

- Homework 1
  - Due Sun, Sep 7