

Ch 5.1.1-2: Leave One Out Cross-validation

Lecture 12 - CMSE 381

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Announcements

Last time:

- Exam

Announcements:

- Exam 1 grades.... hopefully soon
- HW #4 will be posted later next week.
 - ▶ Changed Deadline! Due Sunday Oct 12.

CMSE381_F2025_Schedule : Schedule

	M	9/22	Project Day & Review		
	W	9/24	Midterm #1		
12	F	9/26	Leave one out CV	5.1.1, 5.1.2	
13	M	9/29	k-fold CV	5.1.3	
14	W	10/1	More k-fold CV	5.1.4-5	
15	F	10/3	k-fold CV for classification	5.1.5	
16	M	10/6	Subset selection	6.1	
17	W	10/8	Shrinkage: Ridge	6.2.1	
18	F	10/10	Shrinkage: Lasso	6.2.2	HW #4 Due Sun 10/12
19	M	10/13	PCA	6.3	
20	W	10/15	PCR	6.3	
	F	10/17	Review		
	M	10/20	Fall Break		
	W	10/22	Midterm #2		
21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/28
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	

Covered in this lecture

- Validation Set
- LOO CV

Section 1

Validation set

What's the problem?

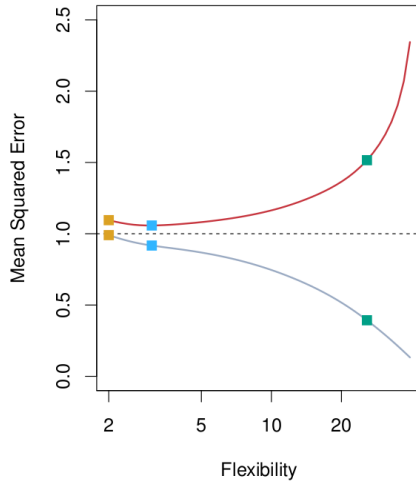
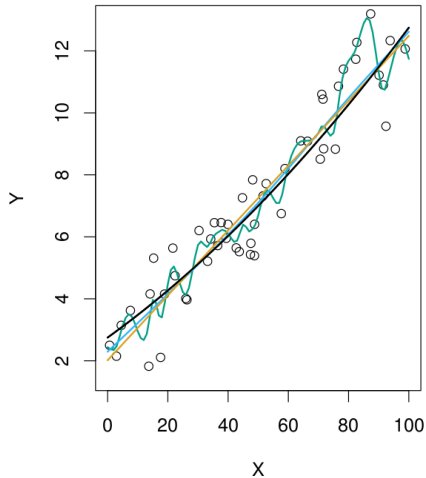
- How well is my ML method doing? *Model Assessment*
- Which method is best for our data?
- How many features should I use? Which ones? *Model selection*
- What is the uncertainty in the learned parameters?

Training Error vs Testing Error

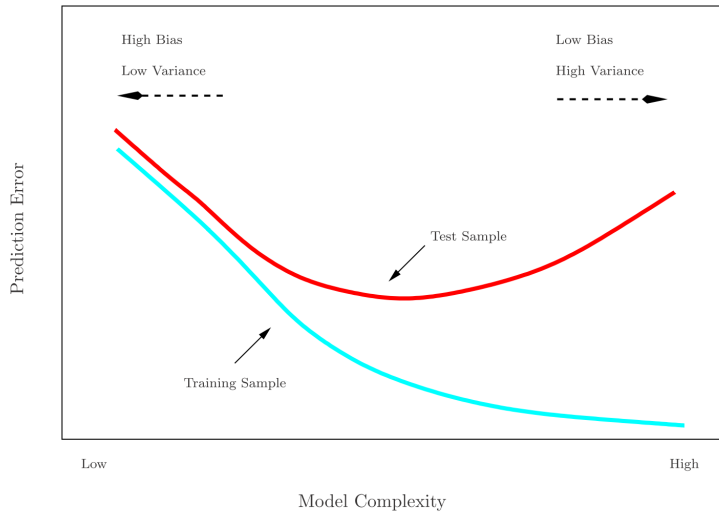
Training Error

Testing Error

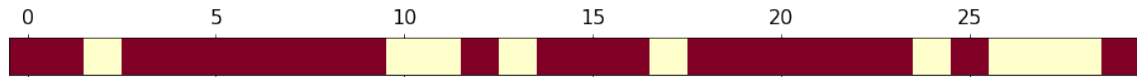
Throw-back



Model tradeoffs

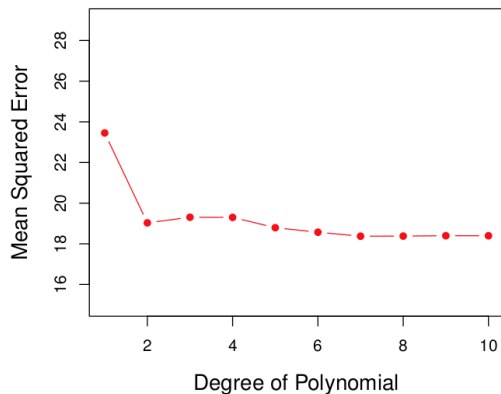


Validation set approach



- Divide randomly into two parts:
 - ▶ Training set
 - ▶ Validation/Hold-out/Testing set
- Fit model on training set
- Use fitted model to predict response for observations in the test set
- Evaluate quality (e.g. MSE)

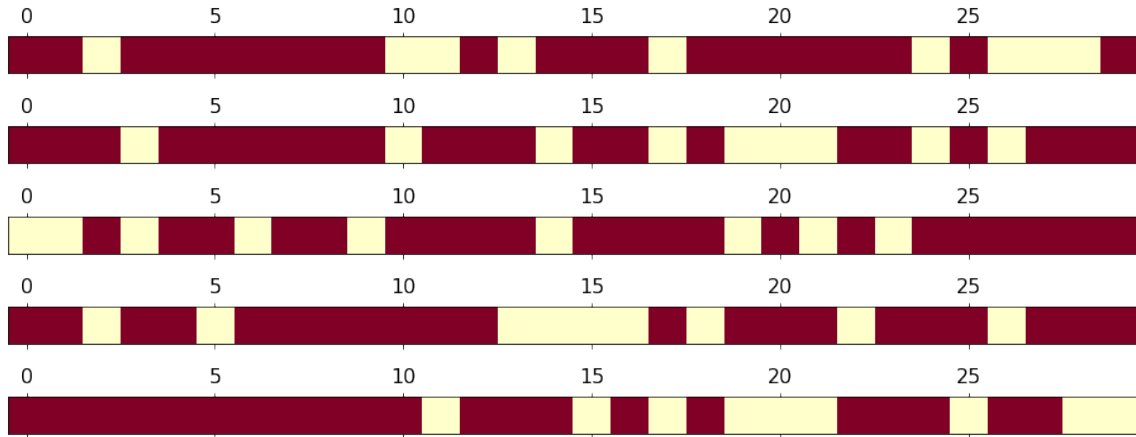
Example with the auto data



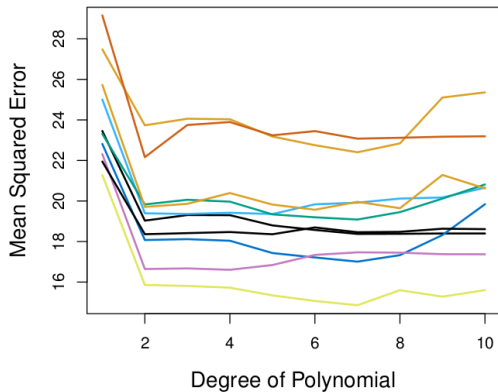
Predicting mpg using horsepower:

$$\text{mpg} = \beta_0 + \beta_1 \text{hp} + \beta_2 \text{hp}^2 + \cdots + \beta_p \text{hp}^p$$

Rinse and repeat



Again example with auto data

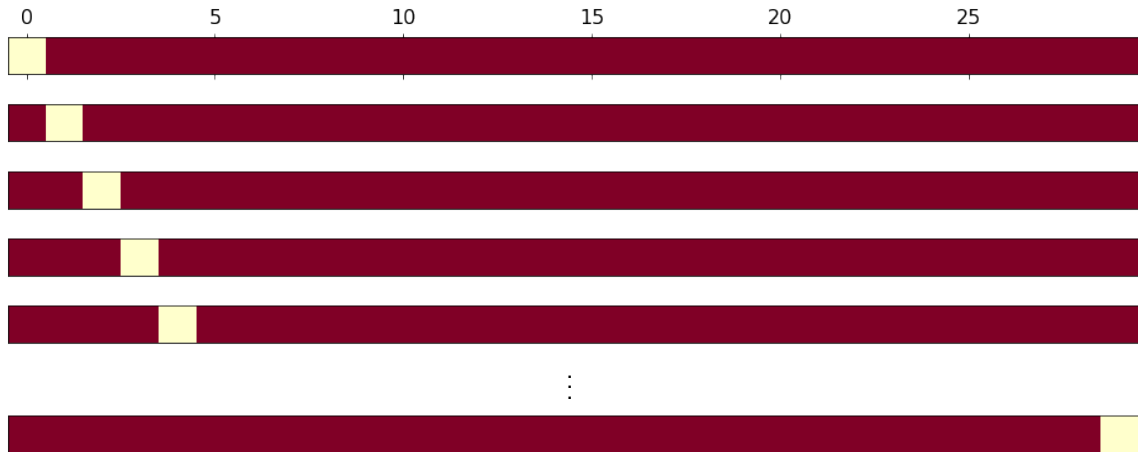


Coding example in jupyter notebook

Section 2

Leave-One-Out Cross-Validation (LOOCV)

The idea



The idea in mathy words

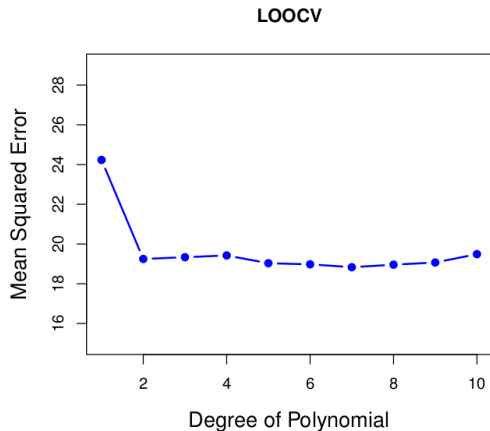
- Remove (x_1, y_1) for testing.
- Train the model on $n - 1$ points:
 $\{(x_2, y_2), \dots, (x_n, y_n)\}$
- Calculate $\text{MSE}_1 = (y_1 - \hat{y}_1)^2$

Return the score:

$$CV_{(n)} = \frac{1}{n} \sum_{i=1}^n \text{MSE}_i$$

- Remove (x_2, y_2) for testing.
- Train the model on $n - 1$ points:
 $\{(x_1, y_1), (x_3, y_3), \dots, (x_n, y_n)\}$
- Calculate $\text{MSE}_2 = (y_2 - \hat{y}_2)^2$
- Rinse and repeat

Again example with auto data



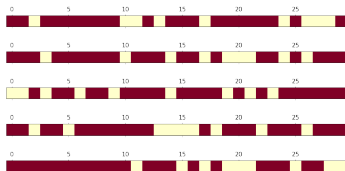
Do the LOOCV coding section

LOOCV Pros and Cons

Advantages:

Disadvantages:

Validation set



LOO-CV



LOO-CV Score

$$CV_{(n)} = \frac{1}{n} \sum_{i=1}^n \text{MSE}_i$$

Next time

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