Ch 5.1.5: k-fold Cross-Validation for Classification

Lecture 15 - CMSE 381

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Announcements

Last time:

- k-fold CV

This lecture:

- CV for classification

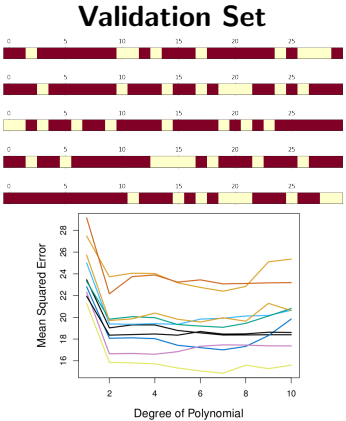
Announcements:

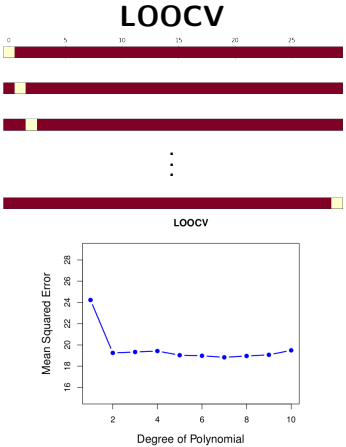
- Homework #4 is posted, Due Sunday (3/2)

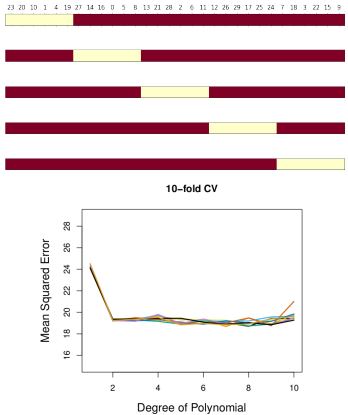
Section 1

Last time

Approximations of Test Error





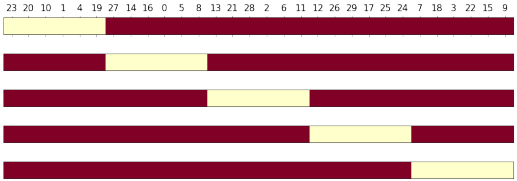


Definition of k-fold CV

- Randomly split data into k-groups (folds)
- Approximately equal sized. For the sake of notation, say each set has ℓ points

- Remove ith fold U_i and reserve for testing.
- Train the model on remaining points
- Calculate $\mathrm{MSE}_i = \frac{1}{\ell} \sum_{(x_j, y_j) \in U_i} (y_j - \hat{y}_j)^2$

- Rinse and repeat



Return

$$CV_{(k)} = \frac{1}{k} \sum_{i=1}^{k} \mathrm{MSE}_i$$

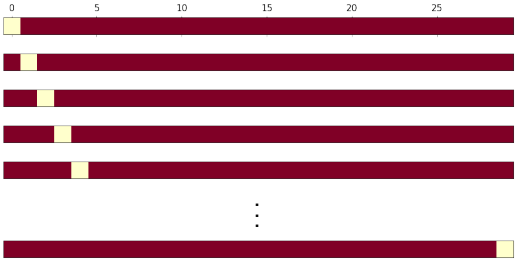
Section 2

CV for Classification

Setup: LOOCV

- Remove ith point (x_i, y_i) and reserve for testing.
- Train the model on remaining points
- Calculate $\mathrm{Err}_i = \mathrm{I}(y_j \neq \hat{y}_j)$

- Rinse and repeat



Return

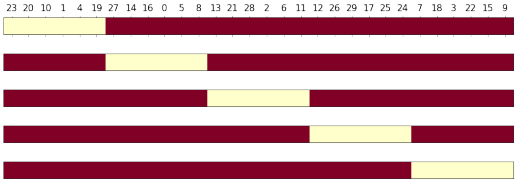
$$CV_{(n)} = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Err}_i$$

Setup: k-fold

- Randomly split data into k-groups (folds)
- Approximately equal sized. For the sake of notation, say each set has ℓ points

- Remove ith fold U_i and reserve for testing.
- Train the model on remaining points
- Calculate $\mathrm{Err}_i = \frac{1}{\ell} \sum_{(x_j, y_j) \in U_i} \mathrm{I}(y_j \neq \hat{y}_j)$

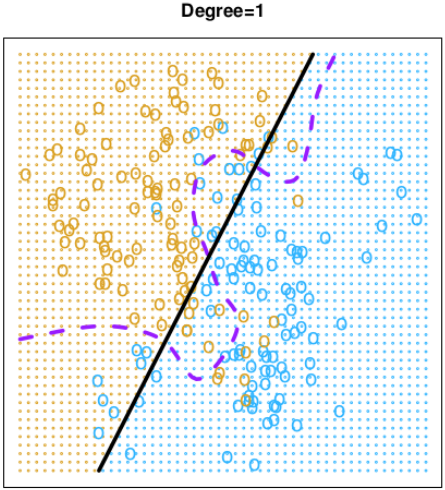
- Rinse and repeat



Return

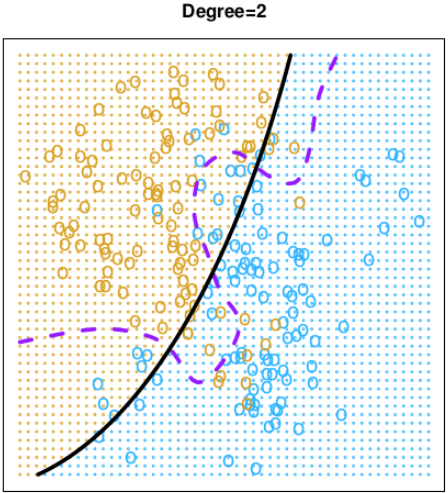
$$CV_{(k)} = \frac{1}{k} \sum_{i=1}^{k} \mathrm{Err}_i$$

Example on simulated data: Linear



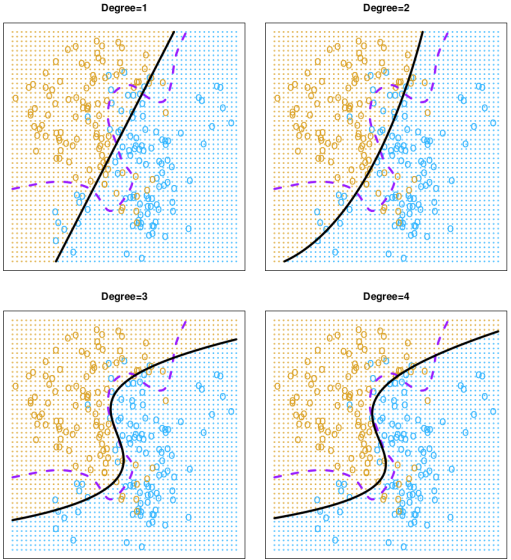
- Purple: Bayes decision boundary.
  - Error rate: 0.133
- Black: Logistic regression
  - $\log(p/(1-p)) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$
  - Error rate: 0.201

Example on simulated data: Quadratic logistic regression



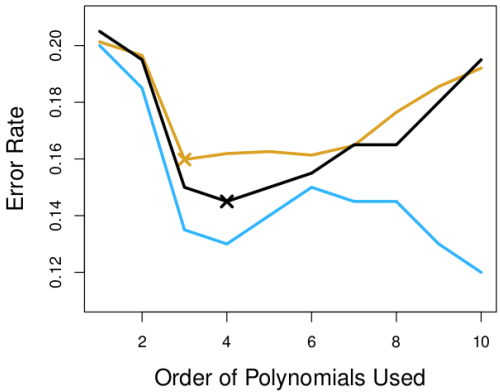
- Purple: Bayes decision boundary.
  - Error rate: 0.133
- Black: Logistic regression
  - $\log(p/(1-p)) = \beta_0 + \beta_1 X_1 + \beta_2 X_1^2 + \beta_3 X_2 + \beta_4 X_2^2$
  - Error rate: 0.197

Example on simulated data: all the polynomials!



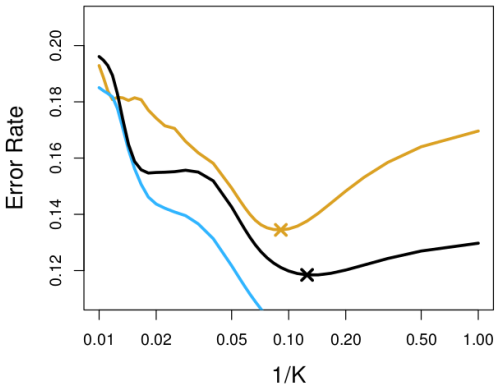
- Purple: Bayes decision boundary.
  - Error rate: 0.133
- Black: Logistic regression
  - Deg 1 Error rate: 0.201
  - Deg 2 Error rate: 0.197
  - Deg 3 Error rate: 0.160
  - Deg 4 Error rate: 0.162

Decide degree based on CV



- Test error (brown)
- Training error (blue)
- 10-fold CV error (black)

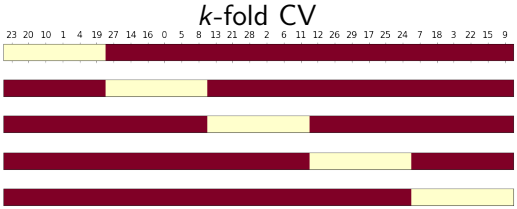
Similar game for KNN



- Test error (brown)
- Training error (blue)
- 10-fold CV error (black)

Coding - k-fold for penguin classification section

TL;DR



$$CV_{(k)} = \frac{1}{k}\sum_{i=1}^{k} \mathrm{MSE}_i$$

Use $k = 5$ or 10 usually

k-fold CV for classification

$$\mathrm{Err}_i = \mathrm{I}(y_j \neq \hat{y}_j)$$

$$CV_{(k)} = \frac{1}{k}\sum_{i=1}^{k} \mathrm{Err}_i$$

Next time

CMSE381_F2025_Schedule : Schedule

	M	9/22	***Project Day & Review***		
	W	9/24	**Midterm #1**		
12	F	9/26	Leave one out CV	5.1.1, 5.1.2	
13	M	9/29	k-fold CV	5.1.3	
14	W	10/1	More k-fold CV	5.1.4-5	
15	F	10/3	k-fold CV for classification	5.1.5	
16	M	10/6	Subset selection	6.1	
17	W	10/8	Shrinkage: Ridge	6.2.1	
18	F	10/10	Shrinkage: Lasso	6.2.2	HW #4 Due Sun 10/12
19	M	10/13	PCA	6.3	
20	W	10/15	PCR	6.3	
	F	10/17	***Review***		
	M	10/20	Fall Break		
	W	10/22	**Midterm #2**		
21	F	10/24	Polynomial & Step Functions	7.1-7.2	HW #5 Due Sun 10/28
22	M	10/27	Step Functions; Basis functions; Start Splines	7.2-7.4	
23	W	10/29	Regression Splines	7.4	